Research Paper

Probabilistic Failure Analysis of Composite Plate due to Low Velocity Impact

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Response under impact, essentially accounts for the most severe damage initiation possibly occurring at the worst possible location in the composite plate. A 3-D explicit dynamic finite element analysis is performed to determine the contact force between the impactor and the target. Quadratic shear failure model is used to predict the failure of a lamina. The uncertainties associated with the properties of composite material, loading condition and assessment of critical stresses affect the failure limit state to a greater extent. If the stresses in the lamina are such that the limit state function \( g(x) \geq 0 \), second order reliability method (SORM) is used to predict the probability of failure (Pf). Monte Carlo simulation (MCS) takes almost 10 times more computational time than SORM. If the impactor of mass is varied from 135 g to 2600 g and velocity of impactor is 1.4 m/s then the probability of failure for cross ply composite plate increases by 99.9 %. Standard deviation of the transverse normal stress \( (S_{13}) \) is most sensitive parameter out of several random parameters involved in the limit state function to influence the Pf.

Key Words: Impact; Damage Initiation; De-lamination; Limit State Function; Probabilistic Failure; Sensitivity

1. Introduction

Laminated composite materials have found extensive applications in the construction of mechanical, aerospace, marine and automotive structures, due to their high specific strength, good corrosion, and fatigue resistance. A prediction of the failure under impact loads is therefore an important topic of research for reliability assurance of composites. Olsson [1] predicted that the impact duration strongly influences the impact response of plates. It is shown that the response type is governed by the impactor plate mass ratio and not by impact velocity and derived a criterion. Small-mass impacts on composite laminates are shown to be more critical than large-mass impacts of the same energy. Aslan et al. [2] performed a study with the transient response of composite laminates subjected to low velocity impact. A drop weight impact load is used for the low velocity impact study and the contact force during impact is accordingly determined. Shi Y et al. [3] estimated that the impact damage of composite laminates in the form of intra- and inter-laminar cracking using stress-based criteria for damage initiation and fracture mechanic techniques to capture its evolution. Donadon et al. [4] predicted 3D failure model of damage in composite structures subjected to multi-axial loading. This formulation enable the prediction of failure initiation and failure propagation by combining stress based, damage mechanics and fracture mechanics approaches within the unified energy based context.

Accurately prediction of the failure under impact loads is an important study for considering the uncertainties of composite material. Considerable work on reliability assessment of composite

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components has been reported. Sriramula et al. [5] reviewed the uncertainties in FRP composites and summarized different stochastic modelling approaches suggested in the literature. Stochastic studies considered uncertainties starting at the constituent level (micro-scale), ply level (meso-scale) or at a component level (macro-scale). Sutherland and Guedes Soares [6] reviewed the work carried out by various authors to investigate the variability arising in the properties of composite materials. Thomas et al. [7] investigated both interactive and non-interactive forms for the failure function. A lower bound limit state for the reliability assessment was found by applying a weakest link criterion to the laminate. An upper bound reliability limit determination could be made by taking an opposite approach by assuming a strong link theory. Cederbaum et al. [8] and Miki et al. [9] used the SORM and Hashin failure criteria to derive the first-ply failure reliability of laminated composite plates subjected to in-plane random loads.

Engelstad and Reddy [10] developed a probabilistic finite element analysis procedure for the prediction of the first-ply failure reliability of composite laminates with the use of the first-order reliability method using Tsai-Wu criterion. Kam and Chang [11] used a phenomenological failure criterion to establish the limit state equation of laminated composite plates for the first-ply failure reliability assessment. Guedes Soares [12] gave a state of the art survey of reliability of composite components and also presented different practical reliability approaches to assess the reliability of composite structures and discussed on various failure criteria. Jeong and Sheno [13] presented a direct simulation approach to assess the first-ply failure reliability of composite structures. They used MCS technique to estimate probability of failure. Ahmad and Gupta [14] used the second order statistics to model the first ply failure. First order reliability method and 2D Hashin’s failure criteria was used to determine the probability of failure and sensitivity analysis of a circular composite plate. Patel et al. [15] used to GRSM coupled with Chang-Chang failure initiation criterion to estimate the probability of failure and sensitivity analysis of a composite beam under ballistic impact.

Keeping in view the above-mentioned survey of the literature, no noticeable work has been attempted where the probabilistic delamination failure of laminated composite plate with scatter of the material properties under low velocity impact is considered. This problem is presently attempted.

The present study investigates the dynamic response of a composite plate subjected to low velocity impact loading. It is showed that the contact force time history for damage delamination initiation model is adopted. The Quadratic de-lamination shear failure criterion for composites is adopted to establish the limit state function in a more realistic fashion. The probabilistic failure behavior is the prime objective of the present study. It is carried out using second order reliability method considering the scatter in the material properties for relevant projectile velocities, ply orientation and boundary condition. SORM is also validated with MCS. Optimum combination of impactor mass and velocity is also carried out for design requirement. The sensitivity of probability of failure with respect to various random parameters has also been investigated. It is an important tool to improve the design of target composites.

2. Behaviour of Composite Plate under Impact Load

Composite plate under study contains eight plies of glass epoxy placed at given orientation [0°/90°/0°/ 90°]. Experimental values and respective statistical properties of glass epoxy have been adopted as reported in the literature [2, 13]. The dimensions of the square plate are 150mm x 150mm x 4.8mm. An impactor of hemispherical shape has radius = 9.0 mm and mass = 2.6kg. The composite plate is meshed with 8 noded brick elements. The plate is clamped along its edges. The impactor of mass is assumed to be rigid. The general contact model is employed to simulate the impact force in ABAQUS. Impact induced damage of composite plate is a probabilistic phenomenon due to wide range of uncertainties arising in material and loading behavior. A typical failure crack propagates initiate as an intra ply matrix crack due to shear or bending. These cracks propagate further in to the interface causing delamination between dissimilar plies. The shear or bending matrix cracks
add further to de-lamination. Quadratic shear failure [16], adopted here, models the de-lamination failure more accurately. The stresses obtained by the finite element software are used in the limit state function to determine the probability of failure.

3. Probabilistic Analysis

Response under impact essentially accounts for the most severe damage initiation possibly occurring at the worst possible location in the composite plate while it is subjected to the highest loads conceivable. In fact, the probability of occurrence of such a scenario is due to large uncertainties arising in the system. Hence, a probabilistic approach is a realistic solution that considers the stochastic variability and distribution of characteristic data of materials. It is needed to account for the uncertainties in composite design, manufacturing and loading conditions. In the present study a composite plate under impact can’t be guaranteed as absolutely safe because of the unpredictability of the loads, uncertainties in the material properties, the use of simplified assumptions in the analysis (which include limitations of the numerical methods used), and human factors (errors and omissions). Nevertheless, the probability of failure is usually required to be within a specified acceptable range for the analysis, design and optimization of a component.

The relevant loads and resistance parameters, essentially random in nature, \(X_i\) and the functional relationship between the response variable \(Z\) (e.g., elastic properties, strength, impactor velocity, stress at a point, deflection etc.) and the random variables \((X_1, X_2, X_3 \ldots \ldots \ldots X_N)\) are described as,

\[ Z(x) = Z(X_1, X_2, X_3 \ldots \ldots \ldots X_N) \]  

A limit state function or performance function is hence defined as

\[ g(x) = Z(x) - Z(0) \]  

where \(Z(0)\) is a limiting/permissible value of \(Z\), an implicit or explicit function of random variables such that \(g(x) = 0\) is a boundary between the failure region \([g(x) < 0]\) and safe region \([g(x) > 0]\). The probability of failure \((P_f)\) is an integral of the joint probability distribution in which \(f(X_1, X_2 \ldots X_N)\) is the joint probability density functions for the random variables \(X_1, X_2 \ldots X_N\), and the integration is performed over the failure region \(X\) where \(g(x) < 0\).

\[
P_f = \iiint f(X_1, X_2, X_3 \ldots \ldots \ldots X_N) \, dX_1 dX_2 dX_3 \ldots dX_N
\]

(3)

This integral is presently computed by the standard Monte Carlo procedure. Depending upon the number of random variables involved and the level of \(P_f\) (usually very small), this integration is performed repeatedly using the reliability code a large number of times to accurately build the response variables’ stochastic characteristics. Although the method is inherently simple, the large numbers of output sets are generated to build an accurate cumulative distribution function of the output variables.

In addition to the CDF of the response, the MCS technique provides additional information regarding the sensitivity of the response to the random variables. The magnitude of the sensitivity factor provides a way to rank the random variables that have the major influence on the uncertainty of the response variable. By controlling the scatter in the more significant variables, the reliability can be improved. In a Monte Carlo simulation, a random value is selected for each of the variables, based on the range of estimates. The model adopted is based on the random values. The result of the model is recorded and the process is repeated. A typical Monte Carlo simulation calculates the model hundreds or thousands of times, each time using different randomly-selected values. These results are used to describe the likelihood or probability of reaching various results in the model. The initial failure of a lamina (mid-plane failure) is governed by exceeding the maximum limit prescribed by a failure criterion adopted and given below.

4. The Limit State Function

Inter-laminar shear stresses are one of the sources of the failure in laminated plates. A quadratic failure criterion [16] is used to describe failure initiation due to these stresses and is given as under:
\[ g(x) = 1 - \left( \frac{S_{33}}{T_{33}} \right)^2 - \left( \frac{S_{13}}{T_{13}} \right)^2 - \left( \frac{S_{23}}{T_{23}} \right)^2 = 0 \] (4)

where \( S_{33}, S_{13} \) and \( S_{23} \) are the inter-laminar normal and shear stresses and \( T_{33}, T_{13} \) and \( T_{23} \) are the corresponding inter-laminar strengths in transverse normal direction and shear directions respectively. Experimentally obtained statistical scatter is also assumed as reported in the literature [13]. The failure initiation is followed by failure propagation in various modes. Cohesive zone formulation is used to study the initiation and propagation. The area under the traction verses relative displacement curves, as given by equation (5) below, represents the energies \( G_{IC}, G_{HC} \) and \( G_{IIIC} \) required for de-lamination in modes I, II and III respectively. The bilinear traction-separation relation as per modes I, II and III are denote to the fracture toughness represented by \( G_{IC}, G_{HC} \) and \( G_{IIIC} \) respectively as given below

\[
\delta_s' \int S_{33} d\delta_s = G_{IC}, \quad \delta_s' \int S_{13} d\delta_s = G_{HC}
\]

and

\[
\delta_s' \int S_{23} d\delta_s = G_{IIIC}
\] (5)

The final relative displacements, \( \delta_s', \delta_s' \) and \( \delta_s' \), corresponding to complete de-cohesion are obtained as: \( \delta_s' = 2G_{IC}/T_{33}, \delta_s' = 2G_{HC}/T_{13} \) and \( \delta_s' = 2G_{IIIC}/T_{13} \).

A limit state in general, is a function of random variables or their functions employed to model the requisite performance of the system under study.

5. Results and Discussion

For numerical study the data is obtained from the literature (2). The respective statistical data adopted as reported in the literature (13). Explicit time domain analysis has been carried out to obtain the response.

(a) Validation Study

It is shown in Fig. 1, the contact force time histories of composite laminates are obtained at low velocity impact using finite element code. Note that the numerically calculated contact force between the impactor and the target has nearly the same values as the experimental results. The results of the same study are presently observed to be smaller but follow the similar trend as published [2]. The difference is arising because of the damage model presently adopted. It is based on damage initiation while Aslan et al. [2] adopted the damage propagation model.

(b) Effect of Impactor Velocity

Fig. 2 shows the contact force time histories of the composite plate numerically obtained with an impactor mass of 2.6 kg striking at different velocities. It is observed that the impactor velocity varied from 1 to 3 m/sec, the peak contact force also increases in linear fashion. The maximum impact force varies linearly with velocity and double the velocity double the contact force.

(c) Cumulative Probability of Failure

Cumulative probability of failure is an important property of the system to optimize the design with respect to statistical properties of random variables to achieve the required reliability level. The probabilistic response of the composite plate in terms of \( P_f \) is significantly influenced by the variation of impactor mass and velocity. Cumulative probability function plots (Figs. 3, 4, & 5) show the estimation of probability of failure against variation of response (Z)
using SORM and MCS respectively. The comparison shows that the accuracy of SORM is significantly close to MCS. MCS required one million cycles to achieve this accuracy. It is computationally very expensive in comparison to SORM. Table 1 showed the probability of mid-plane failure under low velocity impact for different impactor masses and velocities. Due to the design requirement for impactor velocity and mass in terms of Pf may be selected from Table 1 for simply supported plate. It is found that the acceptable range of Pf (1e-3 to 1e-4) for combinations, of impactor mass 2.6 kg and velocity 1.2 m/s as well as similar impactor mass 135 g and velocity 1.8 m/s. It is also observed that the impactor mass vary with 135 g to 2600 g keeping velocity (1.4 m/s) as constant then the probability of failure for cross ply composite plate increases to 99.9%. It means that this combination (impactor mass at 2.6 kg and velocity 1.4 m/s) is not suitable for design requirement. It is also found that the impactor velocity vary with 1.2 m/s to 1.3 m/s and mass of impactor (2600 g) constant then probability of failure increases 67.2%.

(d) Sensitivity Analysis

Sensitivity analysis of limit state function with respect to the changes in the random variables involved in the mathematical model can be apportioned, qualitatively or quantitatively. It is a technique for systematically changing the strength and resistance parameters in a model to optimize the composite design to achieve target reliability. Sensitivity levels ($\psi$) indicate the influence of mean ($\mu$) and deviation ($\sigma$) on probability of failure “$P_f$”. The following equations state the same in general form,

$$\psi_1 = \left(\frac{\partial p}{\partial \mu}\right) (\sigma / p) \quad (6)$$

$$\psi_2 = \left(\frac{\partial p}{\partial \sigma}\right) (\sigma / p) \quad (7)$$

The rate of change of probability of failure for all the random variables diminishes at the required value of the limit state function. It is found that the probability of failure is proportional to the mean value of the stresses (S33, S13 and S23) and scatter of the

Table 1: Probabilistic failure analysis of mid-plane composite plate under impact

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Velocity (m/s)</th>
<th>Probability of Failure (2.6 kg)</th>
<th>Probability of Failure (135 g)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SORM ($P_{f1}$)</td>
<td>MCS ($P_{f2}$)</td>
</tr>
<tr>
<td>1</td>
<td>3.0</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>0.96</td>
<td>0.94</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>0.74</td>
<td>0.69</td>
</tr>
<tr>
<td>4</td>
<td>1.4</td>
<td>0.195</td>
<td>0.187</td>
</tr>
<tr>
<td>5</td>
<td>1.3</td>
<td>0.064</td>
<td>0.063</td>
</tr>
<tr>
<td>6</td>
<td>1.2</td>
<td>31E-4</td>
<td>30E-4</td>
</tr>
<tr>
<td>7</td>
<td>1.1</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
input parameters. It is also observed that the probability of failure is inversely proportional to the mean value of the strengths (T13, T23 and T33) parameters. The scatter of the transverse normal stress direction (S33) is most sensitive parameter to influence the probability of failure. Their influence in terms of their means and standard deviations are shown in Fig. 6. The reliability reduces with the increase in mean shear stress as well as with the increase in its scatter (σ). Design process of the composite plate is accordingly governed to achieve target reliability.
6. Conclusions

The dynamic response of the laminated composite plate in terms of impact force is significantly influenced by the variations of impactor mass and velocity. The impact force is linearly varied with impactor velocities. The limit state function formulated using quadratic de-lamination initiation failure criteria considering three modes of failure. MCS takes almost 10 times more computational time than SORM. Probabilistic design requirement due to the delamination of mid plane Pf (1e-3) is found the impactor mass 135 g and velocity 1.8 m/s is acceptable and similarly the impactor mass 2.6 kg and velocity of impactor is 1.2 m/s. It is observed that the impactor mass vary from 135 g to 2600 g and velocity of impactor (1.4 m/s) then the probability of failure for cross ply composite plate increases 99.9 %. It is also observed that the impactor velocity vary with 1.2 m/s to 1.3 m/s and mass of impactor (2600 g) constant then probability of failure increases 67.2%. Standard deviation of the stress in normal transverse direction in tension (S33) is the most sensitive parameter to influence the probability of failure. Other parameters like inter-laminar shear stresses, normal transverse strength and inter-laminar shear strengths have significant influence on the probability of failure. Systematic changes in the input parameters are governed by probabilistic sensitivity tools to achieve target reliability.

References
