Response of Metallic Plates Subjected to Underwater Explosion

Y PAWAN KUMAR1* and N K GUPTA2**

1Defence Business Unit, Larsen & Toubro Limited, New Delhi 110 020, India
2Department of Applied Mechanics, Indian Institute of Technology, New Delhi 110 016, India

(Received 14 March 2013; Accepted 02 April 2013)

The structural response to various impulsive loads specifically to blast loads in the elasto-plastic region will be in the form of large deformation, crack initiation and tensile tearing. The above failure modes can be predicted reasonably accurately at the design stage based on finite element modelling employing nonlinear shock analysis. In the prediction of structural response beyond elastic limit, both material and geometric nonlinearity are to be considered. The material non-linearity is generally modelled by von Mises yield criteria and associated flow rules with isotropic hardening. The material models such as rigid plastic, elastic perfectly plastic and elasto-plastic can be used to represent the material behaviour. The geometric non-linearity is based on total or updated Lagrangian formulation.

Cole [1] established useful empirical relations to model the underwater explosion (UNDEX) loading, which were the outcome of numerous experimental investigations done by the military agencies. In the present study the UNDEX load profile has been modelled as an exponentially decaying shock wave which varies spatially and transiently. An elasto-plastic model with isotropic hardening, strain rate effects and fracture criterion has been developed was used to predict and establish the failure modes in high strength (HS) and WELDOX Steel rectangular plates under clamped edge conditions. These models were implemented in the non-linear finite element code ABAQUS/Explicit. The present study establishes empirical relationships derived from numerical analysis of stiffened and un-stiffened plates subjected to UNDEX.

Key Words: Plate; UNDEX; Underwater Explosion; Large Deformation

1. Introduction

This study concentrates on establishing the response behaviour of both stiffened and un-stiffened rectangular plane plates of thickness varying from 2 mm to 12 mm when subjected to UNDEX loading with shock factor varying from 0.3SF to 1.74SF. Response of the plates to such an event is function of various parameters which include peak pressure, exponential decay, cavitation loading (secondary and subsequent pulse loading), spatial and temporal variation of shock pulse on the structure, short duration loading and pressure wave propagating as a shock wave. The complexity of the problem in hand is to incorporate the physical phenomenon and simulate numerically the UNDEX event. Systematic research in the area has been reported since World War I. Fox [2] performed experiments on the failure modes in steel plates subjected to underwater blast and compared the failure in air blast events. Later Kiel [3] reported on a spectrum of topics related to underwater explosion event ranging from hull damage, equipment damage to deducing empirical relations for shock wave-pressure time history. Jiang and Olson [4] studied the central deformation of submerged stiffened plates numerically by using a rigid-plastic formulation and also adopted the Hains acoustic approximation which captures the fundamental fluid-structure interaction (FSI) phenomenon neglecting

*Authors for Correspondence: E-mails: pawankumary@gmail.com; nkgupta@iitd.ac.in
**Present Address: Supercomputer Education and Research Centre, Indian Institute of Science, Bangalore 560 012, India
effects such as secondary loading and diffraction, in a different study they also studied a simplified decoupled approach to arrive at workable numerical procedure incorporating FSI by Kirchhoff’s integral equation on the wet surface of circular plates, and the results were found to match with experimental results. Shin and Hooker [5] investigated the damage response of long stiffened circulars cylinders with initial imperfections numerically using a strain hardened mild steel. Rajendran and Narasimhan [6] conducted experiments on HSLA (high strength low alloy) steel circular plates and reported the dynamic radial and longitudinal strains.

The modes of failure were analytically studied and classified by Jones [7, 8] and Nurick [9] using rigid plastic material models on impact on beams subjected to air-blast loading these; these are: (1) Mode I – large deformation (2) Mode II – tensile tearing in outer fibers at the support (3) Mode III – transverse shear failure at the support were observed. Further the Mode II failure was elaborated into Mode II* - partial tensile tearing and Mode II(a) - complete tensile tearing with increased midpoint deflection, the Mode II(c) – central rupture with partial tearing, and concluded that the Mode II and Mode III failures culminated into rupture of plate and were closely interrelated. Recently Ramajeyathilagam and Vendhan [10] conducted underwater explosion experiments and reported the central deformations of MS and HSLA steel plates and brought out the importance of incorporating strain rate in numerical simulation procedures.

In the present work we have used these experiments for validation of the numerical scheme. Stiffened and un-stiffened WELDOX 460E plates were used to study the response of the plates to varying shock factor and plate thickness. The load was modelled as an amplitude curve following an exponential decay with initial peak, varying spatially along the plate surface, typically as seen in an UNDEX event. Applicability of Nurick and Backer [11] empirical equations for large plate thickness and underwater explosive loads has been discussed.

2. Modelling the Explosion Loading

The classical method of treatment to the problem has been approached in modelling the explosion loading. Previously established empirical relationships on the UNDEX blast pressure propagation and decay has been used in the present study. All the referred equations to define the loading phenomenon are relationships to charge mass and the standoff distance for the region of interest [1], the exponential decaying pressure profile is given by the following equation:

\[ P(t) = P_{\text{max}} \cdot e^{-\frac{(t-t_0)}{\theta}}, 0 \leq t \leq \theta \] (1)

where, \( P_{\text{max}} \) and decay constant (\( \theta \)) can be determined by the following relations:

Maximum pressure,

\[ P_{\text{max}} = K_2 \cdot \left( \frac{W^3}{R} \right)^{A_1} \] (2)

Decay constant,

\[ \theta = K_2 \times W^{\frac{1}{3}} \cdot \left( \frac{W^3}{R} \right)^{A_2} \] (3)

In the above equations \( P(t), P_{\text{max}} \) are in MPa and \( \theta, 1, E \) are in microseconds, kPa and k/)m² respectively. In addition to the initial blast loading the UNDEX event involves nonlinearities in fluid medium in the form of cavitation at the fluid solid surface interface. The total pressure at the fluid-structure interaction gets modified due to the relative velocities of the plate and the adjoining fluid; this phenomenon is incorporated in the present study using the Taylors theory as given by the following expression:

\[ p_t = 2p(t) - \frac{2\rho c P_{\theta}}{\sin(\phi)(m - \rho \phi)} \left( e^{\frac{t}{\thetaelta}} - e^{\frac{-t}{\thetaelta}} \right) \] (6)

where,
\[ \varphi = \sin^{-1}\left( \frac{R_0}{R} \right) \] (7)

Table 1 summarizes the constants for the equations for \( P_{\text{max}} \), \( \theta \), I, and E for TNT (Tri-Nitro Toluene) type explosive. These constants are valid when the charge weight (W) is expressed in weight of TNT and stand-off distance (R) is in meters away from the explosive charge. The expressions (1)-(7) are not valid in the immediate vicinity of the explosive but from a distance of 10 times charge diameter [1].

The fluid structure interactions were adopted by using Taylor’s plate theory. The theory calculates the modified pressure field in the vicinity of a reflected wave from a structure and induced velocities in the plate. Underwater explosion can be characterized by “shock factor” which is a function of charge weight, standoff distance, angle of explosion, interference at surfaces, etc. The shock factor (SF) is defined as [1];

\[ SF = \frac{(1 + \cos \theta) \sqrt{W}}{2} \] (8)

where, \( W \) is the charge weight, \( R \) is the charge standoff distance and \( \theta \) is the angle of the shock wave reaching the point under consideration. The charge mass was varied from 0.01-0.79 kgs such that an increasing trend in the shock factors at an interval of 0.5 ranging from 0.3 SF to 1.74 SF, could be obtained.

### 3. Numerical Model Definition

The response of the plate to a spatially and transiently varying shock wave is studied by adopting an elasto-plastic material model with strain rate hardening which is evaluated by progressively plotting the yield surface with the advent of plastic deformation in the material unlike von-Mises surface which has a constant radius.

#### 3.1 Johnson Cook Material Model

Numerical solutions of plastic strain rate is associated with spurious oscillations which can be avoided by adopting damping by first order viscoplastic correction which is shown in the Johnson-Cook model. This model has special capability to handle problems involving intense impulsive loading due to high velocity impact and explosive detonation such as UNDEX loading. The radius of the von-Mises stress envelope is given by:

\[ Y = \left[ A + B \epsilon_p^n \right] \left[ 1 + C \ln \epsilon_p^* \right] \left[ 1 - T_H^m \right] \] (9)

where,

- \( \epsilon_p \) = effective plastic strain
- \( \epsilon_p^* \) = normalized effective plastic strain rate
- \( T_H \) = homologous temperature = \( (T - T_{\text{room}}) / (T_{\text{melt}} - T_{\text{room}}) \)
- \( A, B, C, n \) & \( m \) = material constants

#### 3.2 Johnson Cook Failure Criterion

This failure model adopts the value of equivalent plastic strain at element integration points to cumulatively compute progressive damage and failure of materials. This model is used in conjunction with the Johnson-Cook plasticity model, which is suitable for materials under high strain rate such as typically in UNDEX loads. The material failure is defined by damage parameter (\( \Omega \)) defined as;

\[ \Omega = \sum_{i=0}^{n} \left( \frac{\Delta \epsilon_i}{\epsilon_f} \right) \] (10)

where, \( \epsilon_i \) is an increment of the equivalent plastic...
strain, $\varepsilon_f^c$ is the strain at failure, and the summation is performed over all increments in the analysis. The material failure is assumed to occur when the damage parameter $W > 1$.

4. Results and Discussions

4.1 Response of Flat Un-Stiffened & Stiffened Flat Plates With Varying Shock Factor

This study was conducted on quarter plate numerical models confirming to mechanical properties of WELDOX 460-E. The study was separately conducted on unstiffened plates and on stiffened. The shock factor was varied gradually by increasing the charge mass (W) and standoff distance (R) in steps. Cases studied numerically incorporated Johnson-Cook (J-C) yield and failure criterion. The Johnson-Cook (J-C) constants for WELDOX 460E steel, obtained from [12] have been summarized in Table 2 below.

The study on stiffened and unstiffened plates with incremental shock factors had distinctive humps at shock factors 0.8 SF and 1.65 SF. A further investigation on the results was carried out to reason the humps when the SF was incremental as seen in Fig. 1 (a) and (b). A ratio of weight of explosive to the standoff distance was used to investigate the discontinuity of results obtained. The W/R at 0.8 SF was 0.625 whereas that at 0.6 SF was 0.367 and 0.7 SF was 0.4. The hump at 1.6 SF was at W/R ratio of 3.16 but that at 1.5 SF and 1.65 SF were at 2.7 and 1.35 respectively. It was noted that the humps occurred when the W/R ratio was higher when compared to that at the adjacent shock factors. For a higher W/R ratio the charge mass is either higher or the standoff distance is lower, which causes local failure of the plate due to the proximity of the charge to the target plate. The investigation revealed that when the charge is placed closer to the target plate the tendency to show higher modes of failure than Mode I is seen.

4.2 Comparison of the Central Deformation for Stiffened and Un-Stiffened Plates with Same Mass per Unit Area

An un-stiffened plate of dimension 0.5 x 0.5 m with thickness of 4 mm and a stiffened plate of dimension 0.5 x 0.5 m with plate thickness of 3 mm and stiffeners 0.5 x 0.021 x 0.004 m (6nos) arranged as seen in the Fig. 2 (a) and (b) below.

With similar masses of both the configuration comparative study was conducted on optimized mesh densities for both the analysis as shown in Table 2. Shock factors till 1.2 SF for both the curves are seen to be parallel and above 1.2 SF the un-stiffened plate curve is seen to show higher deformation with increase

<table>
<thead>
<tr>
<th>Table 2: WELDOX 460 E material constants [12]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress (MPa)</td>
</tr>
<tr>
<td>Strain hardening</td>
</tr>
<tr>
<td>Strain rate hardening (S⁻¹)</td>
</tr>
<tr>
<td>Temperature softening (K)</td>
</tr>
<tr>
<td>Fracture strain constants</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B (MPa)</td>
</tr>
<tr>
<td>n</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>$\varepsilon^c$ (S⁻¹)</td>
</tr>
<tr>
<td>$T_r$ (K)</td>
</tr>
<tr>
<td>$T_m$ (K)</td>
</tr>
<tr>
<td>m</td>
</tr>
<tr>
<td>$D_1$</td>
</tr>
<tr>
<td>$D_2$</td>
</tr>
<tr>
<td>$D_3$</td>
</tr>
<tr>
<td>$D_4$</td>
</tr>
<tr>
<td>499</td>
</tr>
</tbody>
</table>
Table 3: Test plate characteristics of density 7850 Kg/m³

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Dimension</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffened Plate</td>
<td>0.5x0.5x0.003 m and with 6 stiffeners of dimension 0.5x0.021x0.004 m</td>
<td>7.86 Kgs</td>
</tr>
<tr>
<td>Un-Stiffened Plate</td>
<td>0.5x0.5x0.004 m</td>
<td>7.85 Kgs</td>
</tr>
</tbody>
</table>

in shock factor. This phenomenon may be seen as the mass participation by the stiffeners in resisting deformation, by restricting growth in the deformation curves with increase in shock factor as seen in Fig. 3. The stiffened panels assumed comparatively smaller deformations as compared to un-stiffened panels due to the load sharing of the stiffeners in their in-plane direction; in-plane direction is the best loading direction for plane members used as stiffeners. The phenomenon of the formation of humps in as explained the previous sections were still found to be congruent in both cases.

4.3 Determination of Load Sharing Between Both Plate Configurations of Same Mass Per Unit Area to Study the Failure Process

Load sharing between the un-stiffened and the stiffened configurations were studied in terms of the equivalent plastic strains (PEEQ) in the model at a

1.74 shock factor (150 gm TNT at 0.1 m standoff). The computation paths for extraction of PEEQ are as shown in Fig. 4 and Fig. 5 for both configurations. In order to study the variations in the PEEQ plots at
the defined paths on both the models the derived plastic strain values for both configurations were plotted together as seen in the Fig 6. It is very evident that although the both configurations had the same mass per unit area, the stiffeners altered the strain flow path in the model very significantly.

The maximum strain magnitude difference in both the configurations was very large as seen in the un-stiffened case the strains are 2.5 and that in the stiffened configuration being 1.0. The strain plot profile of the stiffened plate has two intermediate troughs and a central hump this is due to the combined effect of the loading and the geometric configuration and placements of the stiffeners in the grillage setup. The troughs formed signify the location of two cross stiffeners in the perpendicular direction in the model. The central hump was seen even though the central stiffener was placed in the model because of the fact that the intensity of the UNDEX load applied was maximum at the standoff on the plate and would decay spatially and transiently over the plate surface with epicenter on the center of the plate. A contour plot of the variation of PEEQ along the models in both configurations can be seen in Fig. 7 and Fig. 8.

In both the configurations it is seen that the failure process of the plate was evident to start along the center of the edge of the plate. The strain variation along the central path of the plate has been studied to ascertain the variation of the strains with increase in
the shock factor. Fig. 9 shows the variation of the strains along the center path of the plate (refer inset in Fig. 9) with increase in the shock factors.

The maximum PEEQ values when plotted against increasing shock factor is seen to follow the same trend of curve as in maximum deformation vs. shock factor as seen in Fig. 10. There is a direct correlation between the shock factor and the maximum equivalent plastic strain measured on the plate as seen in the graph below. Hence a linear variation of the results may be safely assumed for the plate response with increasing shock factor, which can in turn be correlated into various combinations of the weight of the TNT charge and standoff distance. The Fig. 10 also suggests that the plate will show higher modes of failure when even a smaller charge is placed near to the plate when compared to that of a larger charge placed far from the plate, even though both the shock factors remain same. The peaks in the Fig. 10 are suggestive of this fact.
The Fig. 10 shows three points A, B and C with their corresponding characteristics mentioned. The point A corresponds to a 55 gm charge of TNT placed at 150 mm standoff distance. Similarly point B corresponds to 45 gm charge of TNT placed at 100 mm standoff distance and that of the point C corresponds to 40 gm charge of TNT placed at 10 mm standoff distance. Points A, B and C correspond to shock factors 0.7SF, 0.8SF and 0.9SF correspondingly. The point B has displayed a higher PEEQ value than that at C although the shock factor of B is lower than that at C. Hence, we can conclude that the contact explosions (when the standoff distance is zero) generally lead to higher modes of failure than that due to non-contact ones with similar shock factors.

4.4 Parametric Study to Develop Empirical Relations from the Numerical Results

The numerical deflections were compared to empirical formulations suggested by Backer and Nurick. The predictions by Backer and Nurick [11] compare favourably with experiments only for small deflections within the range of one half to one plate thickness, as seen in the results of 2mm stiffened plate with shock factors less than 0.9 SF. As the underwater explosions is generally encountered by structures in an event of an unfortunate accident or an intended purpose of destructive force. The general intensities of an attack which is predictable than the case of an accident, the peak pressures are sufficiently high with large impulse values. The numerical simulation on stiffened plates reported in the present study incorporates all the important factors such as strain rate, failure initiation and propagation model in incorporating the large deformation phenomenon of during an underwater explosion. The linear interpolation curves for a 2mm thick plate are as seen in Fig 11. The fitted curves for other thickness were similar with fitting equations as given in Table 3. The numerical results yielded the following set of equations best fit to determine the d/t values for varying impulse loads. The “Y” intercept of the equations refer to the Deflection to thickness ratio and the “X” intercept will be for the total impulse values. These equations have been formulated with the results of a 3D FEA on plates considering strain rate effects, plasticity constitutive model and failure criterion explicitly defined in the analysis.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Plate thickness</th>
<th>Linear best fit equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 mm</td>
<td>y = 0.329x - 47.88</td>
</tr>
<tr>
<td>2</td>
<td>4 mm</td>
<td>y = 0.048x + 0.357</td>
</tr>
<tr>
<td>3</td>
<td>6 mm</td>
<td>y = 0.020x + 0.891</td>
</tr>
<tr>
<td>4</td>
<td>8 mm</td>
<td>y = 0.012x + 0.495</td>
</tr>
<tr>
<td>5</td>
<td>10 mm</td>
<td>y = 0.007x + 0.194</td>
</tr>
<tr>
<td>6</td>
<td>12 mm</td>
<td>y = 0.005x + 0.024</td>
</tr>
</tbody>
</table>

Fig. 11: Total impulse on the plate plotted against d/t for (a) Numerical values (b) Baker and (c) Nurick, all of the calculated and predicted values are for 2mm stiffened plates

5. Conclusion

Plate thicknesses varying from 2 mm to 12 mm at an increment of 2mm were studied and the response to underwater explosions was established. The shock factor varying from 0.3 to 1.74 was adopted in analysis. It was observed that the central deformation of the plate in stiffened and un-stiffened configuration increased with increase in shock factor. With thin plates of thickness 2 mm we observe in the central deformation vs. SF plot showed humps, these humps had an influence to increase the central deformation in the plate when the charge is placed close to the structure, which modifies the behavior of the plate to display modes of failure higher than the Mode I (Large
deformation failure). The stiffened plates displayed similar failure profiles as seen in the un-stiffened case earlier, but displaying reduction in the magnitude of the central deformation. The reduced magnitudes of central deformation in the plates were attributed to increase in the stiffness of the grillage due to introduction of stiffeners placed in-plane with applied load.

Comparative plate study with same mass per unit area, at higher shock factors the un-stiffened plate tends to display higher modes of failure but as due to the inherit stiffness of the stiffened plates the curve is comparatively much flatter at these shock factors. At a given shock factor the strain plots for stiffened and un-stiffened configurations show that the strain configurations at the same time step are less concentrated at the center of the edge of the plate with reduced strain values from 2.54 to 1.74 between un-stiffened and stiffened configurations. The stiffeners provide rigidity in the transverse directions in turn reduce the plastic work done and the corresponding kinetic energies of the plate. Hence, the stiffened configuration is not only efficient in reducing structural damage but also provides a safe foundation for mounted equipment by reducing resultant kinetic energy.

A parametric comparative study was taken up to firstly validate the central deformations obtained in the numerical analysis with the available empirical solutions suggested by Nurick and Backer. The equations proposed by Nurick and Backer for the prediction of the central deformations and the %t were predominantly devised for small deformations of the plate, and were not effectively applicable when the plate deformations exceeded more than one times the thickness of the plate. The present study concludes parametric equations for larger thickness plates subjected to underwater explosive loads.

References