Propagation of Compaction Waves in a Double-Layer Metal Foam Block Subjected to Impact

D KARAGIOZOVA*
Institute of Mechanics, Bulgarian Academy of Sciences, Acad G Bonchev Street, Block 4, Sofia 1113, Bulgaria

(Received 01 March 2013; Accepted 09 March 2013)

The deformation of a double-layer foam block impacting a rigid wall is analysed. Two foam layer configurations are considered in order to reveal the characteristic features of deformation under the condition of decreasing velocity during the impact event. No details of the cellular geometry are analyzed and it is assumed that the foam is a homogeneous material. The dynamic compaction of the foam block is described by one-dimensional models. The models are based on the propagation of a strong discontinuity unloading wave when using the actual experimentally derived stress strain curves for two aluminium based foam: Cymat foams with 9.3 % and 21% relative density. Numerical simulations were carried out to verify the proposed model.

Complex patterns of compactions are revealed depending on the layers sequence. It is demonstrated that the level of compaction strongly depends on the material properties and current velocity. Moreover, it is shown that a secondary compaction of the foam layer is possible not only near to the stationary boundary as usually observed but also at the layer interface when a particular layer sequence is arranged. This phenomenon causes a significant increase of the maximum stress at the layer interface, which can be undesirable for some applications.

Key Words: Aluminium Foam; Double-Layer Block; Impact; Compaction Wave; Secondary Compaction; Strain Distribution

1. Introduction

The response of foams with different characteristics to impact and blast has been studied extensively during the past decade. The continual interest in the behaviour of cellular materials under different loading conditions is mainly related to the ability of these materials to manifest a significant increase of strength and energy absorption enhancement when subjected to an intensive dynamic load.

The compaction mechanism is the major source of the enhancement of the energy absorption capacity of the foam and therefore different methods have been proposed in the literature to model foam compaction. A shock wave propagation model in cellular materials was proposed in Ref. [1] to explain the crush enhancement of wood specimens assuming a rigid perfectly-plastic locking (RPPL) mechanism. Retaining the basic characteristics of the one dimensional shock wave models, more detailed material models were used [2,3] to account for the elastic material properties. An elastic-plastic model with hardening was proposed in [4] while an elastic perfectly-plastic-with rigid locking model was applied in [5, 6]. Although different material models were assumed, these shock wave propagation models use either zero strain or the strain value associated with the strain of the fully locked material. Different definitions of the locking strain were utilised in [4] and [3] but good agreement between the numerical

*Author for Correspondence: E-mail: d.karagiozova@gmail.com
simulations and the corresponding analytical models was demonstrated within the considered ranges of impact velocities as the authors were able to define appropriate densification strains for the analysed materials.

The idea of propagation of compaction waves in foam materials was used to predict the deformation and energy absorption of foam layers used in cladding structures in [7-10] and to simulate shock loading on a structure when using metal foam projectiles [11]. The widely used RPPL model [1] is appropriate and easy to apply for the approximation of the stress-strain characteristic of a cellular material with negligible strain hardening. The densification strain is well defined and the model predicts the response of foam materials particularly well for high velocity impact. Many cellular materials, however, exhibit different degree of strain hardening depending on their topology and density [12-15]. The concept of a predefined densification strain is not applicable for the analysis of these materials and the application of the RPPL model to approximate the material properties leads to an overestimation of their energy absorption capacity [16]. A full compaction within the primary stress wave cannot always be achieved in foam materials, which exhibit strain hardening as shown experimentally in Refs [17, 18]. This is particularly true when the applied velocity is a decreasing function of time [16].

In order to analyse the foam compaction under moderate velocity impact, a uniaxial rigid-linear hardening-locking model was proposed in [19], which can predict compaction strains that are smaller than the locking strain.

Despite the different approximations of the stress-strain curve, the pre-defined locking strain is the common characteristic of the above studies. A model based on a power-law hardening stress-strain characteristic of the foam was used in [18] to estimate the level of strains at compaction due to a constant velocity impact. A more realistic impact scenario when the velocity decreases with time was analysed in [16]. The propagation of compaction waves in metal foams that exhibit strain hardening and are subjected to moderate initial impact was studied. The analysis was based on the propagation of unloading plastic wave of strong discontinuity when using the actual stress-strain curve that can be characterised by a concave plastic loading path. The proposed approach allowed determining a non-uniform strain distribution behind the wave front with sufficient accuracy. A strong dependence of the strains on the impact velocity within the compacted regions was revealed.

All the above studies, which examine the compaction phenomenon in different cellular materials, consider two impact scenarios in homogeneous foam blocks: in the first, a rigid mass strikes a stationary foam block and in the second, a foam block strikes a rigid stationary target. The analysis of layered foam blocks are restricted to the impact or blast loading of sacrificial claddings when the load is applied to a rigid cover plate. In this paper, an impact of a double-layer foam block on a rigid wall is considered in order to broaden the understanding of the dynamic compaction of foam materials under different loading conditions. The approach, which was proposed in [16], is used in the current analysis with the emphasis on the history and final distribution of the strains within the compacted zones, which develop in foam layers. Numerical simulations are carried out to validate the results from the analytical model.

2. Material Characteristics

The materials selected for the present study are aluminium based foams and they differ by densities and consequently by their stress-strain characteristics. The experimentally obtained quasi-static stress-strain curves [20] for Cymat with \( \rho_0 = 253 \text{ kg/m}^3 \) and Cymat with \( \rho_0 = 570 \text{ kg/m}^3 \) are used. These foam types exhibit significant strain hardening together with considerable experimental variation. The material model proposed in [21] for aluminium foam that exhibits hardening was applied to approximate the stress-strain relationship of the Cymat foams (Fig. 1a) with different densities

\[
\sigma(\varepsilon) = \sigma_y + \gamma \frac{|\varepsilon|}{1 - \rho_0 / \rho_b} + \alpha \ln \left\{ 1 - \left( \frac{|\varepsilon|}{1 - \rho_0 / \rho_b} \right)^\beta \right\}^{-1}
\]

(1)

rather than utilizing particular experimental curves due
where \( \varepsilon_y \) is the strain at yield (taken as positive in compression). At certain initial and boundary conditions, for example a suddenly applied constant or increasing normal velocity a 'shock wave' can occur at a high loading rate. The analogy between the shock wave propagation and compaction of cellular materials with negligible strain hardening allowed good predictions to be obtained when a constant densification strain can be defined in advance [1, 3-5, 9, 11, 14].

In the present analysis, the approach proposed in Ref. [16] is used to analyse the compaction of the foam layers. The latter approach is based on the assumption that a plastic unloading wave starts to propagate if the applied load causes stresses, which exceed the elastic limit of the material. This is a wave of strong discontinuity characterised by discontinuous velocity, stress, strain and density on the wave front while the particles behind the plastic wave front experience elastic unloading [23]. An elastic precursor wave starts to propagate ahead of the plastic wave front.

Generally, if a plastic unloading wave propagates from region B to region A (Fig. 1b), the conditions through the front of discontinuity, the conservation of mass and momentum conservation, are

\[
\rho_A (G - V_A) = \rho_B (G - V_B) \quad (3a)
\]
\[
(\sigma_B - \sigma_A) = \rho_B (G - V_B)(V_B - V_A). \quad (3b)
\]

In Eqs. (3), \( \rho_A \), \( V_A \) and \( \rho_B \), \( V_B \) are the characteristic parameters ahead of and behind the wave front, respectively and \( G = 1/f'(\xi) \) is the speed of the wave front. It is assumed that the engineering stress and strain are positive in compression.

Since the elastic strains are much smaller than the plastic strains due to compaction, the elastic strains in the deformed foam material behind the front of the unloading wave can be neglected and a rigid unloading can be assumed. The continuity equation in the region of unloading may be formulated as [24]

\[
\frac{d\rho_B}{dt} = -\rho_B \frac{dV}{d\xi} \quad (4a)
\]
which leads to
\[
\frac{dV}{d\xi} = 0
\]  
(4b)

behind the wave front thus defining a rigid body motion of the compacted region. The particle velocity, \(V\), is independent of \(t\) and equal to the particle velocity of the wave front.

4. Analytical Model for an Impact on a Rigid Wall

Consider a double-layer foam block impacting a rigid wall in normal direction at initial velocity \(V_0\). Two geometric configurations of the layers are considered. In the first scenario, a foam block comprising ‘soft’ foam layer (Cymat1) with thickness \(L_2\) placed at the proximal end and a ‘hard’ foam layer (Cymat2) with thickness \(L_1\) placed at the distal end as shown in Fig. 2a is analysed. In the second scenario (Fig. 2b), a ‘soft’ layer with thickness \(L_1\) is placed at the distal end while a ‘hard’ layer with thickness \(L_2\) is placed at the proximal end. It is assumed that the two foam layers having different densities are perfectly bonded. The deformations of the constituent layers are calculated from the displacement of the free end of the foam block, \(u_1\), and displacement \(u_2\) of the interface between the foam layers. Due to the different foam density and consequently the foam strength, different pattern of block compaction can develop. Analytical models for the two impact scenarios are presented below.

4.1 Softer Foam Layer Placed at the Proximal end of the Block (Scenario 1)

4.1.1 Deformation Phase 1: After the initial contact between the foam block and the wall, a compaction wave starts to propagate in the proximal layer. The characteristic variables just behind the wave front in this layer are
\[
V_{2B} = 0, \varepsilon_{2B} = \varepsilon_2(t), \sigma_{2B} = \sigma_2(t), \rho_{2B} = \rho_{02}/(1-\varepsilon_2)
\]  
(5a-d)

while just ahead of the wave front
\[
V_{2A} = V_2(t), \varepsilon_{2A} = \varepsilon_2(t), \sigma_{2A} = \sigma_2(t)
\]
\[
= \sigma_{Y2}, \rho_{2A} = \rho_{02}/(1-\varepsilon_2) = \rho_{02}\n\]  
(6a-d)

where \(\sigma_{Y2}\) and \(\sigma_2^d\) are the yield stress and dynamic stress of the proximal layer taken positive in compression, \(\varepsilon_{2B}\) and \(\varepsilon_2\) are the corresponding strains. A compacted region with thickness \(h_2(t)\) forms in the proximal foam layer while the distal layer remains undeformed due to the higher strength and moves at velocity \(V_2(t)\). This phase of deformation continues until time \(t = t_1\) when the compaction wave in the proximal layer reaches the interface between the layers.

Taking into account the mass conservation law within the compacted region \(h_2(t)\), the mass per unit area of the compacted region with non-uniform density can be expressed as a function of the initial foam density of the proximal layer \(\rho_{02}\), as
\[
\int_0^{h_2(t)} \rho_2(\xi) d\xi = \rho_{02} (h_2(t) + u_2(t))
\]  
(7)

where \(u_2\) is the displacements of the layer interface (Fig. 2a).

During the time interval \(0 \leq t \leq t_1\) (Phase 1) the following equations of motion with respect to \(V_2(t), h_2(t)\) and \(u_2(t)\) are obtained
\[
\frac{dV_2}{dt} = -\rho_0L_2 + \rho_{02}L_2 - \rho_{02}(u_2 + h_2)
\]  
(8a)
\[
\frac{dh_2}{dt} = -G_3(\varepsilon_2(V_2)), G_3(\varepsilon) = -[V_2(1-\varepsilon_2)]/[\varepsilon_2],
\]  
(8b)
\[
\frac{du_2}{dt} = V_2(t).
\]  
(8c)

Eqs (8) are solved with initial conditions
\[
V_2(0) = V_0, h_2(0), u_2(0) = 0.
\]  
(9a-c)

In Eqs (8) \(G_3\) is the speed of the compaction wave in the proximal layer and the strain jump on the wave front is a function of the corresponding velocity jump, which varies with time and wave front position along the x axis.

4.1.2 Deformation Phase 2: The second phase of the foam block compaction commences at \(t = t_1\) when new compaction waves start to propagate from the
interface in opposite directions at speeds \( G_1 \) and \( G_2 \) in the distal and proximal layer, respectively. At this time instance, the thickness of the proximal layer is \( h_2(t_1) \). It can be shown that the strain values within this layer are very similar at \( t = t_1 \) [16] but they depend on the applied velocity.

A compacted region with thickness \( h_1 \) forms in the distal layer while a secondary compaction occurs in the proximal layer for \( t \geq t_1 \). Note that the initial thickness of the proximal layer during the second phase of deformation is \( L_2^* = h_2(t_1) \) and it depends on the deformation history during Phase 1. In order to facilitate the analytical solution it is anticipated that the proximal layer is a cellular solid with a uniform initial density

\[
\rho_{02}^* = \frac{\rho_{02}}{(1 - \varepsilon_2(t_1))}.
\]  

(10)

The 'new' material of the proximal layer has an yield stress \( \sigma_{y2}^* = \sigma_2(t_1) \). An example for a stress-strain curve of a material, which undergoes a secondary compaction is shown in Fig. 1a by a dashed line. During Phase 2, the proximal foam layer can be considered as a stationary block while the distal layer can be viewed as a moving block. The assumption of rigid unloading (Eq. (4b)) implies that the layer interface and the compacted regions in the two layers move at a common velocity \( V_2(t) \). The undeformed part of the distal layer (and consequently the free end of the entire foam block) moves at velocity \( V_1(t) \) while the part of the proximal layer ahead of the secondary compaction wave is at rest. The characteristic variables just behind the wave front in the distal layer are (Fig. 2a).

\[
V_{1B} = V_2, \epsilon_{1B} = \epsilon_1(t), \sigma_{1B} = \sigma_1^d(t), \rho_{1B} = \rho_{01}/(1 - \varepsilon_1)
\]

(11a-d)

while ahead of the wave front the corresponding values are

\[
V_{1A} = V_1, \epsilon_{1A} = \epsilon_{y1}, \sigma_{1A} = \sigma_{y1}, \rho_{1A} = \rho_{01}/(1 - \varepsilon_{y1}) = \rho_{01}.
\]

(12a-d)

The characteristic variables just behind the secondary compaction wave front in the proximal layer are

\[
V_{2B} = V_2, \epsilon_{2B} = \epsilon_2^*, \sigma_{2B} = \sigma_2^*, \rho_{2B} = \rho_{02}/(1 - \varepsilon_2^*)
\]

(13a-d)

while just ahead of the wave front

![Fig. 2: Schematic of a double-layer foam block impacting a rigid wall; (a) Scenario 1 – a high density foam layer placed at the distal end of the block; (b) Scenario 2 – a high density foam layer placed at the proximal end of the block](image-url)
\[ V_{2A} = V_{y,2}^* \]
\[ \varepsilon_{2A} = \varepsilon_{y,2}^* \]
\[ \sigma_{2A} = \sigma_{y,2}^* \]
\[ \rho_{2A}^* = \rho_{02}^* \frac{1 - \varepsilon_{y,2}^*}{\varepsilon_{y,2}^*} \approx \rho_{02} \tag{14a-d} \]

where \( V_{y,2}^* = \sigma_{y,2}^*/(\rho_{02} \varepsilon_{02}^*) \), \( \varepsilon_{02}^* \) is the elastic wave speed; \( \sigma_{y,1}^* \) and \( \sigma_{y,2}^* \) are the yield stresses in the distal and proximal layer, \( \sigma_1^d \) and \( \sigma_2^p \) are the corresponding dynamic stresses; \( \varepsilon_2^* \) is the plastic strain in the secondary compaction where \( \varepsilon_2^*(t_1) = 0 \). The superscript * is referred to the characteristics of the ‘new’ material resulted from the initial compaction of the proximal layer during the first deformation phase. In Eqs (11)-(14) the strain jumps on the wave fronts are again functions of the corresponding velocity jumps, \( [\varepsilon_1] = \varepsilon_1([V_1]) = \varepsilon_1([V_1 - V_2]) \), \( [\varepsilon_2] = \varepsilon_2^*([V_2]) \) and they vary with time and wave front position along the \( x \) axis.

The stress jump on the wave front in the distal layer is obtained from Eq. (3b) as
\[ \sigma_1^d = \sigma_{y,1} + \rho_{01} \frac{(V_1 - V_2)^2}{\varepsilon_1} \tag{15} \]

while the speed of the compaction wave relative to the layer interface \( G_1 \) is defined as
\[ G_1 = \frac{1 - \varepsilon_1}{\varepsilon_1} (V_1 - V_2) \tag{16} \]

The speed of the compaction wave in the proximal layer and corresponding stress jump are obtained as
\[ G_2^* = V_2^* / \varepsilon_2^* \text{ and } \sigma_2^p = \sigma_{y,2}^* + \rho_{02}^* \frac{V_2^2}{\varepsilon_2^*} \tag{17a,b} \]

when using Eq. (3a) and (3b) together with Eqs (13a)-(13d) and (14a)-(14b).

Taking into account the mass conservation law within the compacted regions \( h_1(t) \) and \( h_2^*(t) \), the mass per unit area of the compacted regions with non-uniform density can be expressed as functions of the corresponding initial density of the foam layers \( \rho_{01} \) and \( \rho_{02}^* \), as
\[ \int_0^{t(t)} \rho_1(\zeta) d\zeta = \rho_{01} \left( h_1(t) + u_1(t) - u_2^*(t) \right) \tag{18a} \]
\[ \int_0^{t(t)} \rho_2^*(\zeta) d\zeta = \rho_{02}^* \left( h_2^*(t) + u_2^*(t) \right) \tag{18b} \]

where \( u_1 \) and \( u_2^* \) are the displacements of the free end of the foam block and displacement of the layers interface during the second phase of deformation, respectively (Fig. 2a).

It is assumed that at the \( t = t_1 \), the contact forces per unit area at the interface are equal and they are larger than the yield stress of both materials, so that
\[ \sigma_2^p(t_1) = \sigma_1^d(t_1) \tag{19} \]

Therefore, the initial value of the velocity \( V_2(t_1) \) for deformation Phase 2 is obtained from the equation
\[ \sigma_{y,2}^* + \rho_{02}^* \frac{V_2(t_1)^2}{\varepsilon_2^*} = \sigma_{y,1} + \rho_{01} \left( \frac{(V_1(t_1) - V_2(t_1))^2}{\varepsilon_1} \right) \tag{20} \]

Velocities \( V_1(t) \) and \( V_2(t) \) decrease with time due to the energy absorbed by the foam. After time \( t \), the compaction waves have travelled distances \( h_1(t) \) and \( h_2^*(t) \) within the distal and proximal layer, respectively. This phase of deformation continues until time \( t = t_1 \) when the compaction of the distal layer ends, \( V_1(t_1) \) becomes equal to \( V_2(t_2) \) and the distal layer together with the compacted part of the proximal layer continue to move as a rigid body at velocity \( V_2(t) \) thus deforming further the proximal layer.

Assuming that the interfacial pressure between the distal and proximal foam layer is \( p \), during the time period, \( t_1 \leq t \leq t_2 \) the common velocity \( V_2 \) can be obtained from the equations
\[ \rho_{02}^* \left( h_2^* + u_2^* \right) \ V_2 = p - \sigma_1^d \tag{21a} \]
\[ \rho_{01} \left( h_1 + u_1 - u_2^* \right) \ V_2 = \sigma_1^d - p \tag{21b} \]
The conservation of momentum with respect to the distal layer leads to

$$\frac{d}{dt}(V_1 - V_2) = -\frac{\sigma_{11}}{\rho_0_1\left(L_1 - (h_1 + u_1 - u_2^*)\right)} + \dot{V}_2.$$  \hspace{1cm} \text{(22)}

Using the Eqs (15), (17b) and (20, 22) the following equations of motion with respect to $V_1(t)$, $V_2(t)$, $h_1(t)$, $h_2^*(t)$, $u_1(t)$ and $u_2^*(t)$ for $t_1 \leq t \leq t_2$ are obtained

$$\frac{dV_1}{dt} = -\frac{\sigma_{11}}{\rho_0_1\left(L_1 - (h_1 + u_1 - u_2^*)\right)} + \dot{V}_2$$  \hspace{1cm} \text{(23a)}

$$\frac{dV_2}{dt} = -\frac{\sigma_{11} - \sigma_{22}^*}{\rho_0_1\left(h_1 + u_1 - u_2^*\right)} + \rho_0_2\left(h_2^* + u_2^*\right)$$  \hspace{1cm} \text{(23b)}

$$\frac{dh_1}{dt} = G_1(\varepsilon_1(V_1 - V_2)), \quad \frac{dh_2^*}{dt} = G_2^*(\varepsilon_2^*(V_2)) - V_2$$  \hspace{1cm} \text{(23c, d)}

$$\frac{du_1}{dt} = V_1(t), \quad \frac{du_2^*}{dt} = V_2(t)$$  \hspace{1cm} \text{(23e-f)}

where $\sigma_{22}^*$, $\sigma_1$, $G_2^*$ and $G_1$ are defined by Eqs (15b), (14), (15a) and (16) respectively.

The initial conditions for Eqs (23) are

$$V_1 = V_1(t_1), \quad V_2(t_1) = f(V_1(t_1)), \quad h_1(t_1) = h_2^*(t_1) = 0, \quad u_1(t_1) = u_2^*(t_1) = 0$$  \hspace{1cm} \text{(24a-e)}

where $f(V_1(t_1))$ is obtained from Eq (20) for the particular properties of the foam layers at $t = t_1$.

### 4.1.3 Deformation Phase 3: The equations of motion during the third phase of deformation ($t \geq t_2$) are obtained with respect to the velocity $V_2$, thickness of the compacted zone, $h_2^*$, and displacement, $u_2^*$, of the layers interface as [16]

$$\frac{dV_2}{dt} = -\left[\sigma_{y2} + \rho_0^* \left(\varepsilon_2^*(V_2)\right)\right] \frac{1}{\rho_0 L_1 + \rho_0^* (u_2^* + h_2^*)} V_2^2$$  \hspace{1cm} \text{(25a)}

$$\frac{dh_2^*}{dt} = G_2^\ast (\left[\varepsilon_2^*(V_2)\right]) - V_2(t), \quad G_2^\ast (\varepsilon_2^*) = \frac{V_2^2}{\left[\varepsilon_2^*\right]}$$  \hspace{1cm} \text{(25b)}

$$\frac{du_2^*}{dt} = V_2(t)$$  \hspace{1cm} \text{(25c)}

The initial conditions for Eqs (25) are

$$V_2(t_2) = V_1(t_2), \quad h_2^*(t_2) = h_2^*, \quad u_2^*(t_2) = u_2^*$$  \hspace{1cm} \text{(26)}

where $h_2^*$ and $u_2^*$ are the thickness of the compacted zone in the proximal layer and displacement at the layer interface attained at the end of the second phase of deformation. The displacement of the free end of the foam block for $t \geq t_2$ is

$$u_1(t) = u_2^*(t) + (u_1(t_2) - u_2^*(t_2))$$  \hspace{1cm} \text{(27)}

## 4.2 Softer Foam Layer Placed at the Distal End of the Block (Scenario 2)

### 4.2.1 Deformation Phase 1: Let us assume that the time for the propagation of the primary compaction wave in the proximal foam layer with thickness $L_2$ (Fig. 2b) is $t_1$. The characteristic variables just behind and in front of the wave front in this layer are defined by Eqs (5, 6). The equations of motion of the double-layer foam block during the time interval $0 \leq t \leq t_1$ are obtained with respect to the velocities $V_1$, $V_2 = V_1 - V_2$ displacements $u_2, u_3 = u_1 - u_2$ and thickness of the compacted zone, $h_2$ as [10]

$$\frac{dh_2}{dt} = G_2(\left[\varepsilon_2(V_1 - V_2)\right]), \quad G_2(\varepsilon_2) = \frac{(V_1 - V_2)(1 - \varepsilon_2)}{\varepsilon_2}$$  \hspace{1cm} \text{(28a)}

$$\frac{dV_1}{dt} = -\left[\sigma_{y1} + \rho_0 L_1 + \rho_0^* (u_2 + h_2 - u_3)\right]$$  \hspace{1cm} \text{(28b)}

$$\frac{dV_3}{dt} = \left[\sigma_{y1} + \rho_0 \left(\frac{V_2^2}{\varepsilon_2(V_3)} - \sigma_{y2}\right)\right] \frac{1}{\left\{m + \rho_0^* \left(L_2 - (u_2^* + h_2 - u_3)\right)\right\}}$$  \hspace{1cm} \text{(28c)}

\[
\frac{du}{dt} = V_1(t), \quad \frac{du_3}{dt} = V_3(t), \quad V_2 = V_1 - V_3, u_2 = u_1 - u_3.
\]

(28d-g)

In Eqs (28), \(\sigma_{\gamma_1}\) and \(\sigma_{\gamma_2}\) (\(\sigma_{\gamma_1} < \sigma_{\gamma_2}\)) are the yield stresses of the materials of the distal and proximal foam layer, respectively, \(h_2\) is the thickness of the compacted region of the foam behind the wave front in the proximal foam layer and \(m_1\) is the mass per unit area of the compacted region behind the wave front in the distal foam layer. It is anticipated that \(m_1\) can be neglected during this time interval due to the very small thickness of the compacted region in the lower density foam placed near to the interface. Equations (28) are solved numerically with initial conditions

\[
V_1(0) = V_0, V_3(0) = 0, h_2(0) = 0, u_1(0) = 0, u_3(0) = 0
\]

(29a-c)

At \(t = t_1\), the compaction wave in the proximal foam layer reaches the interface between the two foam materials. For \(t > t_1\), a compaction wave propagates through the distal foam layer forming a compaction zone with thickness \(h_1\). The characteristic variables just behind the wave front in this layer are

\[
V_{1b} = 0, \varepsilon_{1b} = \varepsilon_1(t), \sigma_{1b} = \sigma_{\gamma_1}(t), \rho_{1b} = \rho_{\gamma_1}/(1 - \varepsilon_1)
\]

(30a-d)

while just ahead of the wave front

\[
V_{1a} = V_1(t), \varepsilon_{1a} = \varepsilon_1, \sigma_{1a} = \sigma_{\gamma_1}, \rho_{1a} = \rho_{\gamma_1}/(1 - \varepsilon_1) = \rho_{\gamma_1}
\]

(31a-d)

During the second deformation phase, the proximal foam layer does not deform further due to its higher strength attained during the first phase of deformation. The equations of motion with respect to \(V_1\) and \(h_1\) become

\[
\frac{dV_1}{dt} = -\frac{\sigma_{\gamma_1}}{\left[L_1 - (u_1 + h_1 - \bar{u}_1)\right] \rho_{\gamma_1}}
\]

(32a)

\[
\frac{dh_1}{dt} = G_1 \left[\frac{\varepsilon_1(V_1)}{\varepsilon_1}\right], G_1(\varepsilon_1) = \frac{\varepsilon_1}{[V_1]/[\varepsilon_1]} \]

(32b)

\[
\frac{du_1}{dt} = V_1(t)
\]

(32c)

where \(G_1\) is the wave propagation speed. Equations (32) are solved numerically with initial conditions

\[
V_1(t_1) = V_1(t_1), \quad h_1(t_1) = 0, \quad u_1(t_1) = \bar{u}_1
\]

(33a-c)

where \(\bar{u}_1\) is the value obtained at \(t = t_1\) according to Eqs (28).

One can see that Eqs (8), (23), (25), (28) and (32) are implicit with respect to \(V_1\) and \(V_2\) due to the strains dependence on these velocities so that the solution of the above equations would require an iterative procedure. Nevertheless, a simplified approach was successfully applied in [16] to solve similar equations taking into account the unique relationship between the strains and stresses of the analyzed materials. The velocity jumps on the wave fronts in the distal and proximal foam layers \([V_1]\) and \([V_2]\), respectively, can be obtained from the corresponding equations as

\[
[V]_\eta = \left(\left[\sigma\right]_\eta, [\varepsilon]_\eta/\rho_{\gamma_\eta}\right)^{1/2}
\]

(34)

where the subscript \(\eta\) stands for layers 1, 1* (material for the secondary compaction) and 2. Making use of the unique stress-strain relationships of the foam materials, the functions \([V]_\eta = F_{\varepsilon\eta}(\varepsilon_\eta)\) given by Eq. (34) can be inverted to give \(\varepsilon_1 = \varepsilon_1([V]_1), \quad \varepsilon_2 = \varepsilon_2([V]_2)\) and \(\varepsilon_2 = \varepsilon_2^*([V]_2)\) [22], or

\[
[\varepsilon]_\eta = F_{\varepsilon\eta \eta}(\varepsilon_\eta).
\]

The latter functions are calculated numerically at discrete points \(\varepsilon_{\eta,i+1} = \varepsilon_{\eta,i} + \Delta \varepsilon_\eta, \Delta \varepsilon_\eta = 0.005\) for all analyzed materials. The best fit approximation for the virgin materials is obtained by exponential functions with the following general expression [16]

\[
[\varepsilon]_\eta = F_{\varepsilon\eta}(\varepsilon) = a_\eta \exp\left(b_\eta [V]_\eta\right) + c_\eta \exp\left(d_\eta [V]_\eta\right) + e_\eta \exp\left(f_\eta [V]_\eta\right) + g_\eta
\]

(35a)

where coefficients \(a_\eta, b_\eta, c_\eta, d_\eta, e_\eta\) and \(g_\eta\) are characteristic constants for Cymat1 and Cymat2 (Table 1). The function \([\varepsilon]_\eta = F_{\varepsilon\eta}(\varepsilon_\eta)\) for the material, which undergoes a secondary compaction
Table 1: Coefficients in Eqs (35a) and (35b) for the analysed foam materials

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Cymat1</th>
<th>Cymat2</th>
<th>Cymat1*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$ (kg/m$^3$)</td>
<td>253</td>
<td>570</td>
<td>680</td>
</tr>
<tr>
<td>$a$ (s$^{-1}$)</td>
<td>3.04568e+1</td>
<td>3.79094</td>
<td>2.95274e-3</td>
</tr>
<tr>
<td>$b$ (s$^{-1}$)</td>
<td>5.89877e-2</td>
<td>7.79516e-2</td>
<td></td>
</tr>
<tr>
<td>$c$ (m/s$^{-1}$)</td>
<td>-3.07681e+1</td>
<td>-3.90185</td>
<td></td>
</tr>
<tr>
<td>$d$ (s$^{-1}$)</td>
<td>5.85293e-2</td>
<td>7.68194e-2</td>
<td></td>
</tr>
<tr>
<td>$e$ (s$^{-1}$)</td>
<td>-6.08614e-1</td>
<td>-7.03149e-1</td>
<td></td>
</tr>
<tr>
<td>$f$ (s$^{-1}$)</td>
<td>6.61025e-3</td>
<td>7.05050e-3</td>
<td></td>
</tr>
<tr>
<td>$g$ (s$^{-1}$)</td>
<td>9.22978e-1</td>
<td>8.13878e-1</td>
<td></td>
</tr>
</tbody>
</table>

(Cymat1*) is best approximated by the exponential function

$$\varepsilon = F \left[ V \right] = b_n - (b_n - a_n) \exp \left( -c_n \left[ V \right]^{d_n} \right)$$

where coefficients $a_n$, $b_n$, $c_n$, and $d_n$ are characteristic constants for the ‘new’ material. Values of these coefficients are given in Table 1 for the particular loading case. The functions defined by Eqs (35a,b) are used in Eqs (8), (23), (25), (28) and (32) to facilitate explicit expressions in these equations with respect to $V_1$ and $V_2$. Eq. (35) can be obtained in an analytical form if the material stress-strain relationship can be approximated, for example, by a simple power-law function [18].

5. Numerical Model

Numerical simulations were carried out to verify the one-dimensional model of the compaction of a double-layer foam block due to an impact on a rigid wall. Two foam block configurations are modelled. In the first scenario, a foam block comprising Cymat1 foam layer with thickness $L_1 = 150$ mm placed at the proximal end, which is in contact with the wall, and Cymat2 foam layer with $L_1 = 22.2$ mm placed at the distal end is considered. In the second scenario, the Cymat1 foam block with $L_1 = 150$ mm is placed at the distal end while Cymat2 block with $L_2 = 22.2$ mm is placed at the proximal end. The foam blocks have a cylindrical shape with diameter of 100 mm and a tie connection is defined between the layers. An initial impact velocity $V_0$ is prescribed to the entire block. Due to the symmetry of the problem a quarter of the foam block was modelled. The velocities $V_1(t)$ and $V_2(t)$ and displacements $u_1(t)$ and $u_2(t)$ are recorded at the free end of the block and at the interface between the two layers, respectively.

The crushable foam model with volumetric hardening available in ABAQUS/Explicit was used for the foams. In this model, the yield surface evolves in a self-similar fashion and the shape factor $\alpha$ is computed using the initial yield stress in uniaxial compression, $\sigma_c^0 = \sigma_y$, the initial yield stress in hydrostatic compression, $\sigma_e^0$, and the yield strength in hydrostatic tension, $\sigma_r$:

$$\alpha = 3k/\sqrt{(3k_i + k)(3 - k)}$$

Tensile data for Cymat foams reported in Ref. [25] showed that the maximum tensile strength of this material was similar to the initial compression strength, so that values of $k = k_i = 1$ were used in the present analysis.

Solid elements were used to model the foam when an automatic meshing was applied in the (y, z) plane with a maximum element length equal to the element thickness in the x direction. The mesh sensitivity analysis was carried out with respect of the variation of the distances of propagation of the compaction waves, thicknesses of the compacted zones, $h_1(t)$ and $h_2(t)$, longitudinal displacements, $u_1(t)$ and $u_2(t)$, and nominal strains. It was established that the mesh refinement with element thickness smaller than 2 mm has only a marginal effect on the analyzed variables. Elements with thickness of 1.25 mm in the x-direction were used in the present analysis in order to assure a sufficient number of data points in the thinner layer.

6. Results and Discussion

6.1 Impact Scenario 1 – A High Density Foam Layer Placed at the Distal End of the Block

In this section an analysis is carried out on the impact of a double-layer foam block when the high density
foam layer is placed at the distal end as shown in Fig. 2a. The proximal layer is 150 mm thick and it is made of Cymat1 ($\rho_0 = 253$ kg/m$^3$) while a 22.22 mm thick layer made of Cymat2 foam (570 kg/m$^3$) is placed at the distal end of the block. The foam block is impacting a rigid wall with initial velocity $V_0 = 125$ m/s. The velocity attenuation due to the energy absorption is shown in Fig. 3a where the thick lines correspond to the analytical model predictions. During the first phase of deformation, compaction occurs only in the proximal layer while the higher density layer moves as a rigid body. Therefore, the velocity of the free end, $V_1$, and velocity of the layer interface, $V_2$, are equal. At $t = t_1 = 0.8$ ms the compaction wave in the proximal layer reaches the layer interface. Strains between $\varepsilon_{2,\text{MAX}} = 0.656$ at $t = 0$ and $\varepsilon_2(t_1) = 0.628$ occur in the compacted zone with thickness $h_2(t_1) = 52.83$ mm.

Due to the small variation of the strains within the compacted zone it is anticipated that a constant density, $\rho_{02}^{*}$, of the proximal layer can be considered during the second phase of deformation when a secondary compaction of this layer develops; $\rho_{02}^{*} = \rho_{02}(1-\varepsilon_2(t_1)) = 608$ kg/m$^3$. The new foam material is defined by the stress-strain curve shown in Fig. 1a by a dashed line. The second deformation phase commences at $t = t_1$ when two compaction waves start to propagate from the interface in opposite directions in the two layers. The initial value of the velocity at the interface is $V_2(t_1) = 30.5$ m/s. Velocities $V_1(t)$ and $V_2(t)$ decrease during the second deformation phase, which lasts until these velocities become equal at $t = t_2 = 0.97$ ms (Fig. 3a).

The third phase of deformation commences at $t = 0.97$ ms when the distal foam layer has stopped to deform but the remaining kinetic energy is absorbed by the further compaction of the proximal layer. The final distributions of the strains in the compacted zones of the layers predicted by the analytical model are compared with the numerical results in Fig. 3b where a reasonable agreement is observed. The strains due to the secondary compaction of the proximal foam layer are shown in this figure by a dashed line.

An illustration of the strain distributions close to the end of the first phase of deformation is shown in Fig. 4a at $t = 0.84$ ms (the initial size of the block is shown as a rectangular box). One can see that no deformations occur in the distal foam layer during this phase. The final strain distribution in the foam block is shown in Fig. 4b. The secondary compaction, which propagated from the layer interface towards the rigid wall in the proximal layer, is well distinguished.

Due to the unique stress-strain relationship of foam materials exhibiting strain hardening it is possible to estimate the maximum stresses that occur for the different degree of compaction. Maximum values of the stresses within the compacted zones as predicted by the analytical model are shown in Fig. 5 for the impact scenario 1 with initial velocity $V_0 = 125$ m/s. It is evident that an initial stress enhancement of 6.5

---

**Fig. 3:** Comparison between the predictions of the analytical model and results from the numerical simulations for impact scenario 1, $V_0 = 125$ m/s, $l_1 = 22.2$ mm, $\rho_{01} = 570$ kg/m$^3$ and $L_1 = 150$ mm, $\rho_{02} = 253$ kg/m$^3$; (a) Velocity attenuation; (b) Final strain distributions in the foam layers.

**Fig. 4:** Illustration of the compacted zones for impact scenario 1, $V_0 = 125$ m/s, $l_1 = 22.2$ mm, $\rho_{01} = 570$ kg/m$^3$ and $L_2 = 150$ mm, $\rho_{02} = 253$ kg/m$^3$; (a) $t = 0.84$ ms, (b) $t = 1.08$ ms.
MPa occurs at the $t = 0$ in the foam layer, which is in contact with the rigid wall, but significantly higher stress of 11.3 MPa develops at the layer interface due to the secondary compaction.

### 6.2 Impact Scenario 2 – A High Density Foam Layer Placed at the Proximal End of the Block

In this scenario, the distal layer is 150 mm thick and it is made of Cymat1 ($\rho_0 = 253$ kg/m$^3$) while the 22.22 mm thick proximal layer is made of Cymat2 foam ($570$ kg/m$^3$) (Fig. 2b). The foam block is impacting a rigid wall with initial velocity $V_0 = 125$ m/s.

A simpler pattern of deformations occurs for this layer arrangement in comparison with scenario 1. During the first phase of deformation, compaction occurs mainly in the proximal layer. Relatively small deformations are observed from the numerical simulations within an extremely narrow region in the distal layer near the foam interface. These strains are caused by the strength difference of the layers but they are neglected in the analytical model during the first phase. The velocity attenuation is shown in Fig. 6a where the thick lines represent the analytical predictions. It is evident that the interface velocity, $V_2(t)$, decreases rapidly and vanishes at the end of the first deformation phase. A comparison between the displacement-time histories predicted by the analytical model and results from the numerical simulations is shown in Fig. 6b. Relatively large strains develop in the proximal layer during the first phase as shown in Fig. 7a.

The high strength induced by the compaction of the proximal layer prevents this layer from further compaction during the second deformation phase. Compaction occurs only in the distal layer which has a smaller density. The magnitudes of the compaction strains depend on the velocity that occurs at the beginning of the second phase. A comparison between the final strain distributions in the foam layers predicted by the proposed model and numerical results is shown in Fig. 7a. Considerably larger strains develop in the distal layer in comparison with the strains in the
proximal layer. An illustration of the final strain distributions is shown in Fig. 7b.

Making use of the unique stress-strain relationships for the analysed foam materials, the maximum stress values within the compacted zones are calculated by the analytical model and they are shown in Fig. 8. In this impact scenario, the maximum stress of 19.54 MPa occurs in the proximal foam layer at the initial contact with the rigid wall while considerably lower stresses develop in the distal layer due to the lower strength of this material.

![Graph showing stress-strain relationship](image)

Fig. 8: Maximum stresses within the compacted zones as predicted by the analytical model, impact scenario 2, \( V_0 = 125 \text{ m/s}, L_1 = 150 \text{ mm}, \rho_{\text{air}} = 253 \text{ kg/m}^3 \) and \( L_2 = 22.2 \text{ mm}, \rho_{\text{air}} = 570 \text{ kg/m}^3 \)

The reasonable agreement between the analytical and numerical results presented in Sections 6.1 and 6.2 suggests that the proposed analytical model can capture the essential features of the foam compaction for these impact scenarios. No assumption for a pre-defined compaction strains is made, instead, they are obtained as a part of the solution being material and velocity dependent. Complex patterns of compactions are revealed depending on the layer sequence. Moreover, it is shown that a secondary compaction of the foam layer is possible not only near to the stationary boundary but also at the layer interface.

7. Conclusions

The deformation of a double-layer foam block impacting a rigid wall is analysed in the paper. It is assumed that the considered class of foam materials can be modelled as a homogeneous material which exhibits strain hardening. The uniqueness of the stress-strain relationship allowed the formulation of a one-dimensional model of compaction wave propagation in a double-layer foam block when assuming two layer configurations. The model of compaction is based on the propagation of a strong discontinuity unloading wave through the foam when no assumption for a pre-defined densification strain is made.

Numerical simulations are carried out to verify the proposed analytical model. The observed reasonable agreement between the analytical and numerical results suggests that the proposed model can capture the essential features of the foam compaction for the analysed impact scenarios. Complex patterns of compactions are revealed depending on the sequence of layers. It is demonstrated that the level of compaction strongly depends on the material properties and current velocity. Moreover, it is shown that a secondary compaction of the foam layer is possible not only near to the stationary boundary as usually observed but also at the layer interface when a particular layer sequence is arranged. This phenomenon causes a significant increase of the maximum stress at the layer interface, which can be undesirable for some applications.

References

24. Lautrup B *Physics of continuous matter, exotic and everyday phenomena in the macroscopic world*. IOP Publishing Ltd., 2005