Impact Simulation of Rocks under SHPB Test

T CHAKRABORTY*
Department of Civil Engineering, Indian Institute of Technology Delhi, New Delhi 110 016, India

(Received 16 January 2013; Accepted 26 March 2013)

In the present study, an attempt has been made to understand the impact induced stress-strain response of rocks up to 100/sec strain rate through numerical simulation of uniaxial and triaxial split Hopkinson pressure bar (SHPB) tests. The finite element (FE) software Abaqus has been used with explicit time integration scheme to simulate the SHPB tests on rocks. The modelled FE geometry of the SHPB test consists of a steel incident bar, the rock sample and a steel transmission bar. High strain rate material constitutive models have been used to characterise the rock sample. The impact of striker bar on the incident bar is simulated by applying an impact induced incident stress pulse on the incident bar. The analysis results have been compared with the laboratory test data collected from the literature. The numerical simulation of SHPB test on rocks resulted in good correlation with the experimental results. It is observed from the simulation results that the peak axial stress of rocks increases with increasing stiffness of rocks and increasing confining pressure.

Key Words: Finite Element; Impact; Numerical Simulation; Rock; Split Hopkinson Pressure Bar Test

Introduction

Blast and impact resistant design of underground civil infrastructure in rock, e.g. railway stations, powerhouse caverns, ammunition storage structures and energy storage caverns require characterisation of the surrounding rock under impact loading. The split Hopkinson pressure bar (SHPB) has been widely used to experimentally study the response of materials under impact loading. After Hopkinson, Kolsky and Lindholm’s early work [1-5], the split Hopkinson pressure bar has gradually become a widely accepted facility to test the mechanical properties of materials under dynamic loading at high strain rate, e.g. 10²-10⁴/sec [6-12]. The SHPB setup usually consists of a striker bar, an incident bar and a transmission bar with all bars having the same diameter and cylindrical shape. The sample is sandwiched between the incident and the transmission bars; the striker bar hits end of the incident bar in the axial direction with an initial velocity of V₀.

In rock mechanics, the SHPB test has been used to determine the dynamic compressive and tensile strength of rocks [13-27]. Lu et al. [16] showed that the dynamic increase factor (DIF) for rocks, which is defined as the ratio of the dynamic strength to the quasi-static strength in uniaxial compression, increases gradually with strain rate in the low strain rate regime (10⁻⁴-10¹/sec), however turns to rapid increase in the intermediate (10¹-10²/sec) and high (10²-10⁴/sec) strain rate regimes. Lindholm et al. [22] showed that energy required in fracturing basalt rock increased with increasing rate of strain. Li et al. [26] observed that the use of bigger samples is required to reach stress uniformity in heterogeneous rocks. Moreover, reduction in loss of incident wave was observed in the presence of pre-stress. Cho et al. [27] performed dynamic tensile SHPB test on rock and showed that the dynamic tensile strength increases with strain rates.

The SHPB tests performed on rocks reported

*For Correspondence: E-mail: tanusree@civil.iitd.ac.in
in the literature are uniaxial in nature. However, beneath the earth surface, rocks are subjected to high confining pressure. The compressive strength of rock increases and crack density decreases with increasing confining stress. Hence, it is necessary to understand the dynamic behaviour of rocks under multi-axial loading. Blanton [28] performed dynamic triaxial compression test on three rocks to study the effect of strain rates on rocks up to 10/sec strain rate. However, the conventional dynamic triaxial compression tests cannot reach strain rate higher than 10/sec [28]. Nemat-Nasser et al. [29] discussed the setup of triaxial SHPB test. Li and Meng [30] investigated the influence of radial confinement in the SHPB test on concrete like brittle materials and showed that compressive strength increases significantly with increasing confinement pressure as compared to the uniaxial compressive strength. However, triaxial SHPB test has not been performed on rocks in the previous investigations.

Characterisation of rock behaviour under impact loading for wide range of strain rates requires both experimental and numerical simulation of SHPB tests on rock under uniaxial and triaxial conditions. Numerical simulation of SHPB tests on rocks considering the elasto-plastic behaviour of rocks is rather unavailable in the previous studies. Hence, the objectives of the present study are to understand the SHPB simulation technique for rocks and to identify a suitable constitutive model for the impact load simulation on soft rocks. By using the numerical simulation a large number of studies can be performed which otherwise would be impossible to study through experiments involving considerable amount of resource, time and tediousness. In the present investigation, uniaxial and triaxial SHPB tests are simulated using finite element (FE) software Abaqus [31] to study (a) the stress-strain behaviour of rocks and (b) the effect of radial confinement on the peak axial stress of the rocks.

### Numerical Simulation

In the present investigation, finite element code Abaqus is used for modelling rock samples under dynamic compression in SHPB test. Fig. 1 shows a schematic diagram of the SHPB test setup. In SHPB test, when the striker hits the incident bar in the axial direction, a rectangular stress pulse generates, which propagates along the incident bar. When the length of striker is short as compared to the total length of the incident bar and the transmission bar, the peak rectangular incident stress is given by

\[ \sigma_i = \frac{1}{2} \rho_{\text{bar}} c_{\text{bar}} V_0 \]  

and the duration of the stress pulse is given by

\[ \Delta t = \frac{2L_s}{c_{\text{bar}}} \]  

where \( \rho_{\text{bar}} \) is the mass density of the bar; \( c_{\text{bar}} = \)

---

**Fig. 1:** Schematic diagram of SHPB test setup and a typical finite element mesh used in SHPB simulation
\( E_{\text{bar}} \) is the elastic longitudinal stress wave velocity in the bar; \( E_{\text{bar}} \) is the Young’s modulus of the bar; \( V_s \) is the velocity of the striker bar; and \( L_s \) is the length of the striker bar [32]. Transmitted and reflected stress pulses are generated and propagated in the transmission bar and the incident bar, respectively. When there is force equilibrium between the sample’s incident and transmitted sides, the sample is uniformly deformed, and the stress-strain curves of the sample can be determined with one dimensional wave equation assumption given by

\[
e_i = e_i + e_t
\]  

and

\[
s = \frac{A_{\text{bar}} E_{\text{bar}} e_i}{A_{\text{sp}}}
\]

where \( e_i \), \( e_t \) and \( e_r \) are the values of transmitted, incident and reflected signals in the bars, respectively. The parameters \( A_{\text{bar}} \) and \( A_{\text{sp}} \) are the cross-sectional areas of bars and specimen respectively.

In the current study, the incident bar, transmission bar and the sample have been modelled to simulate the SHPB test, as shown in Fig. 1. The length and diameter of the incident and the transmission bars are 2.13 m and 0.915 m, respectively [33]. The sample has both length and diameter of 12.7 mm. The bars were made from high strength maraging VM350 steel (Vasco Pacific, Montebello, CA) and have density, \( \rho = 8100 \text{ kg/m}^3 \); Young’s modulus, \( E = 200 \text{ GPa} \); and bar wave velocity, \( c = 4970 \text{ m/sec} \). The striking velocity of striker bar is considered to be 13.9 m/sec. The bar dimensions, bar material properties and the velocity of the striker bars are taken from [33]. The bars and the sample are placed in the same line. The end of the transmission bar is kept fixed. The impact of the striker bar on the incident bar is simulated by applying an incident stress pulse on the incident bar calculated using equation (1) for time \( \Delta t \) calculated using equation (2).

Three dimensional (3D) solid eight node brick elements with reduced integration and hourglass control (C3D8R) are used for modelling the assembly of incident bar, transmission bar and the rock sample. The C3D8R elements take into account the large mesh distortion without affecting the results due to volumetric locking which is a common problem in large deformation analysis. Fig. 1 shows the 3D meshes of the bars and the rock sample used in the present investigation. The bar and sample meshes are selected through mesh convergence study. There are 77958 elements in the incident bar, 16425 elements in the rock sample and 33489 elements in the transmission bar. The rock sample is meshed using a global seed of 0.003 and bars are meshed using a global seed of 0.05 considered accurate enough for obtaining the strain profile. A general contact algorithm is used to define contact between bars and sample in the SHPB test. A central difference scheme is used to integrate the equation of motion explicitly through time. The method is conditionally stable for time increments (\( \Delta t \)) that are smaller than Courant time limit, \( \Delta t \leq \frac{l}{lc} \), where \( l \) is the smallest element dimension and \( c \) is the speed of sound wave in medium in which it travels. Also, the artificial bulk viscosity is activated to properly represent propagation of the induced compressive stress wave by employing quadratic and linear functions of volumetric strain rates with default values of 1.2 and 0.06, respectively. Gauges in the form of reference points are placed at several locations along the sample length and bars in order to collect the data of incident, reflected and transmitted waves. The contact surface between the incident bar, sample and transmission bar are defined as frictionless and hard. The boundary conditions are applied to bars and sample such that movement in one direction is allowed to simulate the one-dimensional wave propagation which is the basis of SHPB test. The uniaxial simulations are performed for three different rock types, e.g., limestone, weak sandstone and granite. The simulation results for limestone are compared with the experimental data given by Frew et al. [33].

In the present investigation, the crushable foam plasticity model in Abaqus has been used to model soft rocks, e.g., limestone and weak sandstone [34]. The performance of the model has also been tested for hard rock, e.g., granite. The crushable foam plasticity constitutive model can simulate compressive stress induced compaction behaviour.
The yield surface of the model takes an elliptical shape in the mean stress \((p)\) vs. deviatoric stress \((q)\) plane. Inside the yield surface, the behaviour of the rocks remains linear elastic. The elliptical yield surface equation of the model is given by

\[
F = \sqrt{q^2 + \alpha^2 (p - p_0)^2} - B = 0
\]  

(5)

where \(p_0\) is given by \((p_c p_t)/2\); \(p_c\) and \(p_t\) are the yield strength values of the rocks under hydrostatic compression and tension, respectively. The parameter \(B\) is the magnitude of the intercept of the yield surface with the vertical axis for deviatoric stress \(q\); and the parameter \(\alpha\) defines the shape of the yield surface in the meridional plane. The volumetric hardening of the model is defined by providing the experimental data for uniaxial compressive strength with axial strain. The strain rate dependence of rocks is included in the model by defining the increase of dynamic yield strength with respect to the static yield strength, i.e., the dynamic increase factor (DIF) with the increase in strain rate. The constitutive model uses a non-associated flow rule with the plastic potential surface \((G_p)\), given by

\[
G_p = \sqrt{q^2 + \frac{9}{2} p^2}
\]  

(6)

The physical properties of the rocks and yield strength of rocks in compression are given in Table 1. The stress-strain curves for the limestone, weak sandstone and granite are obtained from [33], [35] and [36], and used as input in Abaqus. The DIF parameters under high loading rates used in the present study are given in Table 3. The DIF parameters are obtained in the present study from Li et al. [37].

### Table 1: Properties of rocks

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Mass density, (\rho) (kg/m(^3))</th>
<th>Modulus of elasticity, (E) (GPa)</th>
<th>Poisson’s ratio, (\nu)</th>
<th>Yield strength, (\sigma_y) (MPa)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limestone</td>
<td>2300</td>
<td>24</td>
<td>0.35</td>
<td>68</td>
<td>[33]</td>
</tr>
<tr>
<td>Weak sandstone</td>
<td>2291</td>
<td>13.6</td>
<td>0.35</td>
<td>23</td>
<td>[35]</td>
</tr>
<tr>
<td>Granite</td>
<td>2620</td>
<td>50</td>
<td>0.17</td>
<td>200</td>
<td>[36]</td>
</tr>
</tbody>
</table>

### Table 2: Properties of bars

<table>
<thead>
<tr>
<th>Bar type</th>
<th>Length (L) (m)</th>
<th>Mass density, (\rho) (kg/m(^3))</th>
<th>Modulus of elasticity, (E) (GPa)</th>
<th>Poisson’s ratio, (\nu)</th>
<th>Material</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incident</td>
<td>2.13</td>
<td>8100</td>
<td>200</td>
<td>0.3</td>
<td>High</td>
<td>VM350 [33] steel</td>
</tr>
<tr>
<td>Transmission</td>
<td>0.915</td>
<td>8100</td>
<td>200</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Dynamic increase factor (DIF) for different materials [37]

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Material description</th>
<th>Strain rate ((\dot{\varepsilon})) (1/sec)</th>
<th>Dynamic increase factor (DIF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Limestone</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1</td>
<td>1.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>1.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>1.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>1.95</td>
</tr>
<tr>
<td>2.</td>
<td>Weak sandstone</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td>4.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>5.63</td>
</tr>
<tr>
<td>3.</td>
<td>Granite</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>45</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>145</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>676</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>731</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1023</td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2344</td>
<td>2.47</td>
</tr>
</tbody>
</table>

### Results and Discussions

**Uniaxial SHPB Simulation**

Fig. 2(a) presents the axial stress \((\sigma_a)\) - strain \((\varepsilon_a)\) response of limestone under uniaxial SHPB test simulation. The simulation results are compared with the experimental data given in [33]. It is observed
Fig. 2: Comparison of simulation results with experimental data for limestone

Fig. 3: Comparison of axial stress, incident and transmitted strain in different rocks
from the figure that the simulation results compare reasonably well with the experimental data in the initial part of the stress-strain curve at low strains and in predicting the peak stress. Insignificant mismatch in the results is seen in the strain softening region. The peak axial stress magnitude for limestone is 119.881 MPa. Fig. 2(b) compares the incident ($\varepsilon_i$) and transmitted ($\varepsilon_t$) strain time histories in the SHPB test. The peak strain magnitudes from the simulation results compare well with the experimental data. The time duration of the peak strain is shorter in the case of experiment. The difference between the simulation results and experimental data may be attributed to the application of stress pulse in simulation as opposed to the impact of striker bar in the experiment without pulse shaping.

Fig. 3(a) shows the stress-strain plots of limestone, weak sandstone and granite under uniaxial SHPB test. Maximum peak axial stress is observed in granite, e.g., 145.678 MPa. Limestone and sandstone show 119.881 and 95.19 MPa peak axial

---

![Figure 4](image-url)

**Fig. 4:** Effect of confining pressure on SHPB stress-strain response for different rocks
stresses, respectively. Higher peak stress is observed for rocks with higher stiffness. The initial slope of the stress-strain curves also clearly shows the effect of stiffness of the rocks, i.e. the initial slope decreases with decreasing stiffness of rock. For granite, the loading and unloading paths remain same because granite does not exhibit elasto-plastic response. However, plastic strain is observed in the cases of limestone and weak sandstone which is expected. Figs. 3(b) and (c) compare the incident and transmitted strains for three different rocks. The incident strains are observed to be same in all the three rocks. However, the transmitted strains are different for the three different rocks. The highest amount of transmitted strain is observed in granite whereas the lowest amount of transmitted strain is observed in sandstone. Higher amount of strain in the transmission bar in the case of granite is due to elastic behaviour of granite in SHPB test and thus the absence of plastic dissipation. Plastic dissipation is present in the cases of limestone and sandstone which results in lower amount of strain transmission in these rocks.

\textbf{Triaxial SHPB Simulation}

The triaxial SHPB test is simulated in two steps - (i) application of confining pressure around the rock sample in a static analysis and (ii) simulation of SHPB test in a dynamic explicit analysis. The stress state at the end of static analysis is imported in the dynamic analysis. Figs. 4(a) to (c) show the triaxial SHPB simulation results for limestone, weak sandstone and granite respectively at 0, 5 and 20 MPa confining pressures. The peak axial stress values for all the three rocks at three different confining pressures are noted in Fig. 4. It is observed that the peak axial stress of all the three rocks increased with increasing confining pressure. Also, the initial stiffness of the rocks increases with increasing confining pressure. Fig. 5 summarises the peak axial stress of the rock samples with increasing confinement. The rate of increase of the peak axial stress is higher for soft rocks, e.g. weak sandstone and limestone and lower in hard rock, e.g. granite.

\textbf{Conclusions}

In the present study, an attempt has been made to understand the impact induced stress-strain response of rocks up to 100/sec strain rate through numerical simulation of uniaxial and triaxial split Hopkinson pressure bar (SHPB) tests using the finite element (FE) software Abaqus/Explicit. From these simulations, the following conclusions are drawn:

(1) The simulation results compare reasonably well with the experimental data for the initial part of the stress-strain curve at low strain level and for the peak stress prediction. Insignificant mismatch in the results is obtained in the strain softening region. It may be concluded that the strain rate dependent crushable foam plasticity constitutive model simulates the high strain rate behaviour of rocks with reasonable accuracy.

(2) The initial slope of the stress-strain curve decreases with decreasing stiffness of rock. For granite, the loading and unloading paths remain same because granite does not exhibit elasto-plastic response. However, plastic strain generates in the cases of limestone and weak sandstone which is expected.

(3) The amount of transmitted strain decreases with decreasing stiffness of rocks. Higher amount of
strain in the transmission bar in the case of granite is due to elastic behaviour of granite in SHPB test and thus the absence of plastic dissipation. Plastic dissipation is present in the cases of limestone and sandstone which results in lower amount of strain transmission in these rocks.

(4) The peak axial stress increases with increasing confining pressure on the rock sample. Also, the initial stiffness of the rocks increases with increasing confining pressure.

Acknowledgements

The technical discussions with Dr. Manjit Singh, Director, Terminal Ballistic Research Laboratory (TBRL), Chandigarh had been greatly useful in the numerical simulations and results reported in this manuscript.

References

1. Hopkinson B A method of measuring the pressure in the deformation of high explosives or by the impact of bullets Phil Trans Roy Soc A213 (1914) 437-452
4. Lindholm U S Some experiments with the split Hopkinson pressure bar J Mech Phys Solids 12 (1964) 317-335
6. Nicholas T Material behaviour at high strain rates Imp Dyn (New York: Wiley) chapter 8 1982
8. Franz C E, Follansbee P S and Wright W J New experimental techniques with the split Hopkinson pressure bar (Eds: Berman and J W Schroeder) 8th Int. Conf. on High Energy Rate Fabrication Pres Ves Pip Div ASME (San Antonio, TX, 17-21 June) I (1984)
22. Lindholm U S, Yeakley L M and Nagy A The dynamic strength and fracture properties of dresser basalt Int J Rock


37. Li Q, Crawford J and Hao H Impact and blast effects: Theory, analysis and design Course literature for three day short course, The University of Manchester, UK, 27-29th September 2010.