Research Paper

An Implicit Erosion Algorithm for the Numerical Simulation of Metallic and Composite Materials Submitted to High Strain Rates

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In this paper, we present a general consistent numerical formulation able to take into account strain rate, damage and thermal effects of the material behaviour. A thermomechanical implicit approach for element erosion to model material failure is also presented. This approach can be applied both to ductile fracture for metals, relying on a continuum damage mechanics approach, coupled to different fracture criteria, as well as composite material failure described with either a failure criterion or a progressive damage model. The numerical models will be illustrated by different quasi-static and high strain rate applications for both metallic alloys and composite materials. All these physical phenomena have been included in an implicit dynamic object-oriented finite element code (implemented at LTAS-MNPL, University of Liège, Belgium) named Metafor [1].

Key Words: Numerical Simulation; Finite Element Method; Damage; Fracture; Erosion Algorithm; Implicit Time Integration; Thermomechanical Coupling; Metallic Alloys; Composite Materials

1. Introduction

Advanced high-strength steels, aluminium, titanium alloys are all gaining popularity in automotive and aeronautics applications because they exhibit either a rather large ductility at a high strength level. As a result, these newly developed alloys are considered ideal for crash energy absorption, fatigue and durability of sensitive parts. With proper design strategy, these materials offer a great opportunity for both weight reduction and crashworthiness. In a finite element simulation involving high strain rates phenomena, two main issues have to be taken into account. At first, adequate constitutive equations including strain rate and temperature dependence must be used. Secondly, the study of fast phenomena must be coupled with large strain formulation.

The interest for composite materials has also been increasing exponentially for the last few years. Thanks to their high strength-to-weight and stiffness-to-weight ratios, composites are now widely used in various industry sectors such as automotive industry, civil engineering and ship building. In aeronautics, the design of lighter structures leading to reduced fuel consumption is strongly motivated by economic and environmental concerns. As a result, the proportion of composite parts in aircraft design has been constantly growing during the last few years. However, the conception of structures taking full advantage of the composite weight-saving potentiality strongly depends on the designer’s ability to accurately predict material failure. The mechanical properties of composites are more complex than those of metals because of the heterogeneous and most often anisotropic nature of the former. This is especially true for damage phenomena, which combine several
microscopic mechanisms identified as follows for laminate composites: fibre breakage, matrix cracking, fibre-matrix debonding and delamination [2]. A direct simulation of these mechanisms in numerical analyses of macroscopic structures is impractical. The design of a macroscopic damage model ensuring the desired accuracy at a reasonable computation cost is therefore essential for the simulation of composite structures.

There exist different approaches to simulate composite damage with finite elements. Failure criteria predict brittle failure of the material under static loading [3]. They range from the empirical Tsai-Wu criterion [4], which gives a simple polynomial expression of the failure envelope, to the physically-based criterion of Puck and Schürmann, which makes distinction between fibre failure and inter-fibre fracture [5]. On the other hand, progressive damage models allow a more precise analysis of the damage process and its time evolution, at the cost of increased implementation complexity and computation cost. An example of such models is the meso-model by Ladevèze et al. which gives very accurate results for unidirectional composite laminates [2, 6].

In the present work, the thermomechanical constitutive equations are coupled with an element erosion algorithm, which allows element removal when a fracture criterion is reached. Thus, crack propagation can be simulated for both quasi-static phenomena as well as dynamic problems. This erosion algorithm can be coupled for both metallic alloys models and composite material models as illustrated in the last section. All the algorithmic set up presented here has been implemented in our in-house finite element code METAFORE [1] which is developed in an implicit framework.

2. Constitutive Model for Metallic Materials Submitted to High Strain Rate

For such a material behaviour, the constitutive material law has to take into account strain rate dependence to measure the increase of apparent hardening due to the high strain rate and the thermal softening of the material. A material model, such as the Johnson-Cook’s model [7, 8] is particularly well suited for this kind of behaviour. In a Johnson-Cook model, the yield limit \( \sigma_y \) is given by the expression:

\[
\sigma_y = (A + B(\bar{\varepsilon}^p)^n)^m \left( 1 + C \ln \frac{\dot{\varepsilon}}{\varepsilon_0} \right) \left( 1 - \left( \frac{T - T_{room}}{T_{melt} - T_{room}} \right)^m \right)
\]

(1)

where \( A, B, n, C, \dot{\varepsilon}, \varepsilon_0 \) and \( m \) are material parameters, \( T, T_{room} \) and \( T_{melt} \) are respectively the current, initial and melting temperatures, \( \bar{\varepsilon}^p \) is the equivalent plastic strain and \( \dot{\varepsilon} \) its rate. A failure criterion, which also takes into account strain rate and temperature, can be coupled to this constitutive law, such as, for example, in the case of Johnson and Cook’s failure criterion, the fracture plastic strain [8] is given by:

\[
\varepsilon_f = \left( D_1 + D_2 \exp \left( D_3 \frac{\sigma}{\bar{\sigma}} \right) \right) \left( 1 + D_4 \ln \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0} \right) \left( 1 + D_5 \left( \frac{T - T_{room}}{T_{melt} - T_{room}} \right) \right)
\]

(2)

where \( D_1, D_2, D_3, D_4 \) and \( D_5 \) are material parameters, \( p \) and \( \bar{\sigma} \) respectively the hydrostatic pressure and the stress invariant. Details regarding the implicit time integration of such complex material models, including thermomechanical coupling and damage can be found in [9, 10, 11].

3. Constitutive Model for Composite Material with Damage

The composite material considered in this work is a laminate made of a woven carbon fabric with balanced warp and fill yarns and an epoxy resin. Each ply is assumed to be homogeneous and orthotropic, showing a brittle linear elastic behaviour in the warp and fill directions and inelastic behaviour in shear. In general, the plies are logically assumed to be in plane stress state. However, one of the goals of this work is to study delamination after impact, which means that the plies must be modelled with solid elements. For an elastic orthotropic material, the 3D strain-stress relation in orthotropic axes reads [12]:
where the nine independent parameters are the elastic moduli $E_1, E_2, E_3$ the shear coefficients $G_{12}, G_{23}, G_{31}$ and the Poisson ratio $\nu_{12}, \nu_{23}, \nu_{13}$. For the woven ply considered here, the orthotropic axes 1 and 2 are aligned with the fibres and axis 3 is orthogonal to the ply. Plasticity is introduced as follows: the plasticity surface

$$f(\sigma, \bar{\epsilon}^p) = \bar{\sigma}(\sigma) - \sigma_y = 0$$

is defined by the expressions of the equivalent stress

$$\bar{\sigma} = \sqrt{\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2}$$

which depends in this case on shear strains only. The equivalent plastic strain is defined as:

$$\bar{\epsilon}^p(t) = \int_0^t \sqrt{\left(\dot{\epsilon}_{12}^p\right)^2 + \left(\dot{\epsilon}_{23}^p\right)^2 + \left(\dot{\epsilon}_{31}^p\right)^2}$$

and the yield limit reads:

$$\sigma_y(\bar{\epsilon}^p) = \sigma_y^{el} + K(\bar{\epsilon}^p)\gamma,$$

where $R_o, K$ and $\gamma$ are material parameters.

(a) Failure Criterion

The well-known failure criterion proposed by Tsai and Wu [4] defines a failure envelope that can be written as

$$F_i \sigma_i + F_j \sigma_j = 1,$$

where $F_i$ and $F_j$ are material parameters and $\sigma_i$ is the $i^{th}$ component of the vector $[\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]$.

For 3D orthotropic materials expression (8) reduces to [12]:

$$F_1 \sigma_{11} + F_2 \sigma_{22} + F_3 \sigma_{33} + F_{11} \sigma_{11}^2 + F_{22} \sigma_{22}^2$$

$$+ F_{33} \sigma_{33}^2 + F_{44} \sigma_{23}^2 + F_{55} \sigma_{12}^2 + F_{66} \sigma_{12}^2$$

$$+ 2F_{12} \sigma_{12} \sigma_{22} + 2F_{13} \sigma_{11} \sigma_{33} + 2F_{23} \sigma_{22} \sigma_{33} = 1$$

(9)

Parameters $F_i$ and $F_j$ are functions of the tensile and compression strengths $X^T, X^C, Y^T, Y^C, Z^T, Z^C$ in the orthotropic directions 1, 2 and 3 respectively, and of the shear strengths $S_{12}, S_{23}, S_{31}$:

$$F_1 = \frac{1}{X^T} - \frac{1}{X^C}, F_2 = \frac{1}{Y^T} - \frac{1}{Y^C}, F_3 = \frac{1}{Z^T} - \frac{1}{Z^C}$$

(10)

$$F_{11} = \frac{1}{X^T X^C}, F_{22} = \frac{1}{Y^C Y^T}, F_{33} = \frac{1}{Z^C Z^T}$$

(11)

$$F_{44} = \frac{1}{S_{23}^2}, F_{55} = \frac{1}{S_{31}^2}, F_{66} = \frac{1}{S_{12}^2}.$$ 

(12)

The missing parameters are difficult to identify experimentally and are generally approximated with the formulae

$$F_{12} = -\frac{1}{2} \sqrt{F_{11} F_{22}}, F_{23} = -\frac{1}{2} \sqrt{F_{22} F_{33}},$$

$$F_{31} = -\frac{1}{2} \sqrt{F_{33} F_{11}}.$$ 

(13)

(b) Damage Model

In order to describe damage evolution in the material, one has to consider more elaborate damage models than failure criteria. In composite laminates, damage results from a combination of microscopic mechanisms (fibre or matrix breakage, fibre-matrix debonding, delamination) that are not easily taken into account in a macroscopic material model. In the mesoscopic approach proposed by Ladevèze, Allix et al. [2, 6], a laminate is idealized as a stack of homogeneous plies separated by interfaces. Meso-models were originally developed and validated for unidirectional composites. The woven ply considered
in this work is modelled with a three-dimensional orthotropic law with different properties along the fibres (elastic, brittle failure) and in shear (elastoplastic, progressive damage). The strain energy density reads

\[
W_0 = \frac{1}{2} \left( \frac{\sigma_{11}^2}{E_1(1 - d_{11})} - \frac{2\nu_{12}}{E_1} \sigma_{12} \sigma_{22} - \frac{2\nu_{13}}{E_1} \sigma_{13} \sigma_{33} + \frac{\sigma_{22}^2}{E_2(1 - d_{22})} - \frac{2\nu_{23}}{E_2} \sigma_{23} \sigma_{33} + \frac{\sigma_{33}^2}{E_3} + \frac{\sigma_{12}^2}{G_{12}(1 - d_{12})} + \frac{\sigma_{13}^2}{G_{13}(1 - \lambda d_{12})} + \frac{\sigma_{23}^2}{G_{23}(1 - \lambda d_{12})} \right)
\]

(14)

where \(d_{11}\) and \(d_{22}\) are variables representing damage in warp and fill directions, \(d_{12}\) is the shear damage variable and \(\lambda\) is a material parameter. In order to avoid mesh-dependency issues, a damage evolution law with delay effect is defined,

\[
d_{ij} = \frac{1}{\tau_{\infty}} \left( 1 - e^{-a_{ij} \tau_{\infty}} \right)
\]

(15)

where \(a_{ij}\) and \(\tau_{\infty}\) are delay parameters. The static damage variables \(d_{ij}^s\) are functions of the thermodynamic forces,

\[
Y_{ij} = \frac{\partial W_0}{\partial d_{ij}^s}
\]

(16)

In fibre directions, brittle failure of the fibres leads to the following relations,

\[
d_{11}^s = \begin{cases} 
0 & \text{if } (Y_{11} < Y_{11}^{s+} \text{ and } \sigma_{11} > 0) \text{ or } \\
1 & \text{otherwise} 
\end{cases}
\]

(17)

\[
d_{22}^s = \begin{cases} 
0 & \text{if } (Y_{22} < Y_{22}^{s+} \text{ and } \sigma_{22} > 0) \text{ or } \\
1 & \text{otherwise} 
\end{cases}
\]

(18)

The static shear damage variable is expressed as a function of the equivalent force

\[
Y = \sup_{\tau \leq t}(\alpha_{1} Y_{11}^{s}(\tau) + (\alpha_{2} Y_{22}^{s}(\tau) + Y_{12}(\tau))
\]

(19)

where

\[
Y_{ij}^+ = \begin{cases} 
Y_{ij}^+ & \text{if } \sigma_{ij} > 0 \\
0 & \text{otherwise} 
\end{cases}
\]

(20)

so that the influence on shear damage of stress in fibre direction is accounted for. Static damage is given by the power law

\[
d_{ij}^s = K_0 + K_1 Y_{ij}^0 + K_2 Y_{ij}^0
\]

(21)

where \(K_0, K_1, K_2, \alpha\) and \(\beta\) are material parameters.

(c) Implementation

Damage is introduced into the time-stepping algorithm as follows: let us assume integration from time \(t^n\) to \(t^{n+1}\). The time stepping algorithm is based on an Updated Lagrangian formulation; \(\sigma(t)\) being known, \(\sigma(t^{n+1})\) is calculated thanks to a Newton-Raphson procedure. Inside this non-linear loop, the damage and plastic variables are calculated in an iterative way at each Gauss point. The set of time dependent damage variables, which depends on the damage model, is globally noted \(D(t)\). Plasticity is solved using the radial return approximation [9]: with the strain increment \(\Delta \varepsilon\) given by the Newton-Raphson loop and the damage variables \(D\), one calculates the elastic predictor \(\sigma_{0}(\Delta \varepsilon, D)\) and the normal to the plasticity surface \(\sigma_{n}\). In order to satisfy condition (4), the coefficient \(\Gamma\) has to be determined:

\[
f(\sigma_{n} - \Gamma \sigma_{n}^{0} + \tilde{e}_{p}(\Gamma)) = 0.
\]

(22)

Here is a schematic description of the algorithm. Noting \(V^{0}\) and \(V^{1}\) variable \(V\) at time \(t^n\) to \(t^{n+1}\) respectively, we have to find \(D^1\) and \(\sigma^1\) starting from, \(D^0\), \(\sigma^0\) and the strain increment \(\Delta \varepsilon\):

\[
D^{1,j=0} = D_0
\]

Loop on \(j\) (damage):

\[
\sigma_{ij}^1 = \sigma_{ij}(\Delta \varepsilon, D^{1,j})
\]

\[
\Gamma^{j,k=0} = 0
\]

Loop on \(k\) (plasticity):

\[
f^{j,k} = f(\sigma_{ij}^1 - \Gamma^{j,k} \sigma_{n}^1)
\]

If \(f > f_{\text{max}}\) :
\[ \Gamma^{j,k+1} = -f^{j,k} \left( \frac{\partial F}{\partial \Gamma} \right)^{-1} \]

\[ k = k + 1 \]

\[ D^{j,f+1} = D(\sigma^f_{\nu} - \Gamma^{j,k}\sigma_n) \]

\[ j = j + 1 \]

\[ \sigma^1 = \sigma^f_{\nu} - \Gamma^{j,k}\sigma_n \]

4. Thermomechanical Implicit Algorithm Coupled with Erosion Algorithm

(a) Implicit Staggered Thermomechanical Algorithm

The main feature of this paper is the use of unconditionally stable implicit staggered thermomechanical algorithms including inertia effects [11] to time integrate the conservation equations, instead of the traditional conditionally stable explicit algorithm generally used for impact problems [13]. The algorithm is here enhanced with the introduction of an erosion element method such as the one proposed by Belytschko and Lin [14], but presented here in an implicit time integration framework. This algorithm allows element deletion when a fracture criterion is satisfied inside the element.

When describing fast phenomena, it is important to take into account two effects. The first one is the inertia forces. If inertial terms are often negligible in metal forming problems, they become very important for an accurate description of fast phenomena such as crash and impact simulations. The second important physical effect that is present in metallic materials is the thermal softening due to heat generation by viscoplastic dissipation. In a slow phenomenon, it can be neglected because heat production is limited and heat can be eliminated by conduction and exchange with external environment without much affecting the mechanical response. For fast processes however, the increase in temperature can be high and is concentrated in areas of high strains because almost no conduction or other heat exchange can take place within such a limited amount of time. So a coupled thermomechanical numerical analysis taking into account these two effects is essential to model accurately dynamics problems.

Usually, explicit methods are used for the time integration of such dynamic thermomechanical problems. The advantages of these methods are the ease of implementation and a low memory requirement but they are known to be possibly slow because of numerical instabilities when time steps grow, especially in thermomechanically-coupled simulations. Implicit methods can be more efficient in CPU cost because these methods are unconditionally stable and thus larger time steps can be used. But, due to the iterative solution process, they are more difficult to implement and they require more memory (the stiffness matrix must be computed, stored and inverted at each iteration).

In this paper, the thermomechanical problem is solved thanks to an implicit staggered scheme [15] that consists in an operator split for mechanical and thermal sub-problems. At first, the mechanical problem is solved at fixed temperatures and secondly, the thermal problem (heat equation) is solved for a fixed configuration. The coupling terms are included in the thermal dependencies of material constitutive parameters and in the heat equation resolution. The semi-discretized mechanical equilibrium equation is given by:

\[ M\ddot{x} + F^{int,mec} = F^{ext,mec} \]  \hspace{1cm} (23)

With \( M \) the mass matrix, \( \ddot{x} \) the nodal acceleration vector, \( F^{int,mec} \) the vector of internal mechanical forces and \( F^{ext,mec} \) the vector of external mechanical forces. For the time integration of this equation, the Chung-Hulbert or alpha-generalized algorithm family [16, 17, 18] is selected. This algorithm consists in weighting accelerations and forces between time step \( n \) and time step \( n + 1 \):

\[
\begin{align*}
(1 - \alpha_M)M\ddot{x}_{n+1} + \alpha_MM\ddot{x}_n &= (1 - \alpha_F) \\
\left(F^{ext,mec}_{n+1} - F^{int,mec}_{n+1}\right) + \alpha_F\left(F^{ext,mec}_n - F^{int,mec}_n\right)
\end{align*}
\]  \hspace{1cm} (24)

For an adequate choice of parameters \( \alpha_M \) and \( \alpha_F \), this scheme can be proved to be unconditionally stable and second-order accurate in the linear range while maximizing high frequency dissipation and minimizing low frequency dissipation [16]. It is also able to integrate problems where inertia forces are
negligible. The thermal part of the coupled problem consists in the time integration of the semi-discretized heat equation:

\[ C \dot{T} + \dot{f}_{\text{int,ther}} = f_{\text{ext,ther}} \]  

where \( C \) is the heat capacity matrix, \( \dot{T} \) the nodal temperature rate, \( \dot{f}_{\text{int,ther}} \) the vector of internal thermal forces and \( f_{\text{ext,ther}} \) the vector of external thermal forces. This equation is solved at each time step by mid-point generalized or trapezoidal numerical scheme.

(b) Fracture Erosion Algorithm

The erosion method [14], which consists in erasing the broken finite elements according to a fracture criterion, is the most common technique to propagate a crack through a mesh. However, so far, as developed in [14], it was used only in explicit commercial computer codes (e.g. LS-Dyna or Abaqus). In this work, we extend the erosion method to implicit algorithms. Three possible ways to trigger fracture in a given finite element are proposed in this paper. They correspond respectively to criterion 1, 2 and 3. We use here classical Q4 quadrilateral elements or Q8 hexahedra with constant pressure to avoid locking due to the incompressibility of plastic deformation. Thus, in 2D, there are 4 Gauss points per element, which is another significant difference with respect to classically used elements in commercial explicit codes which use only one Gauss point per element and anti-hourglassing procedures. Using criterion 1, the element is broken if a fracture criterion is satisfied at one single Gauss point of the element; Using criterion 2, the element is broken if a fracture criterion is satisfied in average over all Gauss points of a given element; Using criterion 3 the element is broken if a fracture criterion is satisfied at each and every Gauss points of the considered element.

The major contribution of this paper is thus the application of implicit algorithms coupled with the tearing of structures. Indeed, within the framework of problems including fast dynamics with possible tearing, the algorithms used by the commercial codes are always of the explicit type. The solution presented in this paper allows keeping an equilibrated configuration during the tearing of the structure.

5. Numerical Applications

(a) Tensile Test on Double Metallic Notched Specimen

As a first numerical illustration of our developments, an example of crack propagation using erosion technique coupled to an implicit algorithm is presented. The studied problem is a problem of traction of a square plate, bored of two circular notches placed at two diagonally opposed corners of the plate. The geometry and the loading are presented in Fig. 1 and Table 1. The lower and right sides of the plate are clamped, while the left side is fixed only in the horizontal direction (supports with rollers). The loading is imposed through the vertical displacement of the upper edge of the plate amounting to 1.5mm.

The geometrical data of the test are summarized in Table 1.

The material behaviour is modelled thanks to a linear isotropic hardening law, \( \sigma_y = \sigma_0 + h \varepsilon^p \) where \( \sigma_y \) is the current yield limit and \( \sigma_0 \) and \( h \) are material parameters. Fracture criterion of material is given by [19]:

![Fig. 1: Traction of a double notched square](image)

<table>
<thead>
<tr>
<th>Geometrical data</th>
<th>Value (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate length ( \alpha )</td>
<td>10.0</td>
</tr>
<tr>
<td>Notch radius ( R_C )</td>
<td>1.0</td>
</tr>
<tr>
<td>Superior notch radius ( R_1 )</td>
<td>2.0</td>
</tr>
<tr>
<td>Inferior notch radius ( R_2 )</td>
<td>2.5</td>
</tr>
</tbody>
</table>
\[
\bar{\varepsilon}^p = \int_0^1 \frac{1}{C} \left( 1 + A \frac{\bar{p}}{\bar{\sigma}} \right) d\bar{\varepsilon}^p = 1
\]  

(26)

where \(\bar{p}\) and \(\bar{\sigma}\) are respectively the pressure and the equivalent von Mises stress. \(A\) and \(C\) are material parameters and \(\langle x \rangle = x + 1 \times 1\) are Mac Aulay brackets. Material parameters are summarized in Table 2.

As shown in Fig. 2, differences obtained for the different fracture criteria are significant after the maximum force has been reached i.e. when the material begins to fracture. Such an effect cannot be detected by commercial codes since they only use one Gauss point per element. Indeed, if the type of criterion has low influence at the beginning of the loading and on fracture triggering, curves separate thereafter.

(b) Split Hopkinson Tension Bar Simulation

The second example, still dealing with a metallic material, is a Split Hopkinson Tension bar test, smooth or notched. For the notched specimens, three notch radii are considered: 0.4, 0.8 and 2mm. Loading is imposed through displacement of one of the ends of the bar. Three initial temperatures are considered for each test: 100, 300 and 500°C. The four geometries (axisymmetric modelling), fixations and loadings, are presented in Fig. 3. Tested material is Weldox 460E steel and the material behaviour is modelled by the modified Johnson-Cook model presented in a previous section: Material parameters are taken from [8] and are given in Table 3.

### Table 2: Material parameters for the crack propagation test in a double notched square

<table>
<thead>
<tr>
<th>Material parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (E) (MPa)</td>
<td>180000.0</td>
</tr>
<tr>
<td>Poisson ratio (\nu)</td>
<td>0.28</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>2700.0</td>
</tr>
<tr>
<td>(\sigma^d) (MPa)</td>
<td>443.0</td>
</tr>
<tr>
<td>(h) (MPa)</td>
<td>1690.0</td>
</tr>
<tr>
<td>(A)</td>
<td>3.0</td>
</tr>
<tr>
<td>(C)</td>
<td>1.9</td>
</tr>
</tbody>
</table>

### Table 3: Material parameters for the Weldox 460E steel

<table>
<thead>
<tr>
<th>Thermo-elastic parameters</th>
<th>Johnson-Cook parameters</th>
<th>Fracture parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E) (MPa)</td>
<td>200000.0</td>
<td>(A) (MPa)</td>
</tr>
<tr>
<td>(\nu)</td>
<td>0.33</td>
<td>(B) (MPa)</td>
</tr>
<tr>
<td>(\rho) (kg/m³)</td>
<td>7850.0</td>
<td>(n)</td>
</tr>
<tr>
<td>(C_p) (J/kg.K)</td>
<td>452.0</td>
<td>(m)</td>
</tr>
<tr>
<td>(K) (W/m.K)</td>
<td>47.0</td>
<td>(C)</td>
</tr>
<tr>
<td>(\alpha(K))</td>
<td>1.1e-5</td>
<td>(\dot{\varepsilon}_0) (s-1)</td>
</tr>
<tr>
<td>(T_{mol}(K))</td>
<td>1800.0</td>
<td></td>
</tr>
<tr>
<td>(T_{room}(K))</td>
<td>293.0</td>
<td></td>
</tr>
</tbody>
</table>
Comparisons of force-displacement curves are presented Fig. 4 for the four geometries and the three initial temperatures. Results are compared with experimental data and with numerical results obtained with the explicit LS-DYNA code as presented in [20]. Numerical results, both from explicit [20], as well as present implicit time integration schemes fairly agree with the experimental ones. This validates the presented methodology.

**Tension of a Composite Notched Plate**

A tension test of a rectangular notched plate has been realised. The plate is 6cm long, 2.5cm wide and 3mm high; the notches have a radius of 3mm. One of the extremities is clamped whereas the opposite end is moved at constant speed until the displacement reaches 0.1mm. The laminate is made of 8 plies stacked with orientation sequence [(+45°,0°/90°)₂]₅.

---

**Fig. 4: Force-elongation curves for the 4 geometries and the 3 initial temperatures**
The geometry is meshed by extruding the triangular or rectangular 2D mesh of a ply (Fig. 5), with one element per layer in the extrusion direction. Failure is expected to start at the notches and to propagate towards the centre of the plate, as can be seen in Fig. 6.

Quasi-static tensile simulations are made with a linear material and Tsai-Wu criterion in order to facilitate failure detection. The parameters for the material model are given in Table 4. The mesh is gradually refined by dividing the mesh characteristic length in the plane by the same ratio whereas the number of layers is constant. Fig. 7 shows the total force on the ends of the plate versus the displacement $u_{\text{end}}$. As expected, failure is initiated when the

![Notched plate meshed with hexahedra (left) and pentahedra (right)](image)

![Visualisation of broken elements (white area) in a cross section in the middle of the plate at different time steps (quasi-static simulation, Tsai-Wu criterion)](image)

![Fig. 7: Force-displacement curves of the tension test of the notched plate with different meshes (quasi-static simulation, Tsai-Wu criterion). Global view (left) and refined view (right)](image)

![Fig. 8: Force-displacement curves of the tension test of the notched plate for different time steps (transient simulation, damage model). The dotted line indicates the time when the first mesh elements get broken](image)

![Table 4: Material parameters of the linear model of the composite ply.](table)

<table>
<thead>
<tr>
<th>$E_1$</th>
<th>60.6 GPa</th>
<th>$G_{12}$</th>
<th>4.3 GPa</th>
<th>$\nu_{12}$</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_2$</td>
<td>60.6 GPa</td>
<td>$G_{13}$</td>
<td>2 GPa</td>
<td>$\nu_{13}$</td>
<td>0.05</td>
</tr>
<tr>
<td>$E_3$</td>
<td>8.5 GPa</td>
<td>$G_{23}$</td>
<td>2 GPa</td>
<td>$\nu_{23}$</td>
<td>0.05</td>
</tr>
<tr>
<td>$X^T$</td>
<td>842.5 MPa</td>
<td>$X^C$</td>
<td>419.6 MPa</td>
<td>$S_{12}$</td>
<td>100 MPa</td>
</tr>
<tr>
<td>$Y^T$</td>
<td>842.5 MPa</td>
<td>$Y^C$</td>
<td>419.6 MPa</td>
<td>$S_{13}$</td>
<td>10 MPa</td>
</tr>
<tr>
<td>$Z^T$</td>
<td>18.7 MPa</td>
<td>$Z^C$</td>
<td>60 MPa</td>
<td>$S_{23}$</td>
<td>10 MPa</td>
</tr>
</tbody>
</table>

displacement reaches a value ($u_{\text{end}} = 0.07\text{mm}$) that is nearly independent of the mesh. The cracking then propagates until complete breakdown of the structure.
The force-displacement curves remain close to one another after failure initiation, although they do not converge to a single profile when the mesh is refined.

A high-speed tensile loading of a plate is then simulated with the damage model (14-21). This plate is a 12-ply laminate with stacking sequence [0,45]_3s. It is 180mm long, 32mm wide and 4.4mm thick, and notches have a radius equal to 3mm. One extremity is clamped whereas the other is subjected to a displacement of 2mm in 0.1ms. Mesh elements are removed when one of the damage variables \( d_{11}, d_{22}, \) or \( d_{12} \) reaches a user-defined maximum value, in this case 0.95. The force vs displacement curves for different values of the time step are displayed in Figure 8. The dotted line marks the position where the first mesh elements are broken and removed from the mesh. The time evolution of the stress does not show an excessive sensitivity to the time step, even after the first elements are broken, thanks to the progressive damage model. With a failure criterion only, no convergence with respect to the time step could be observed after the breaking point.

6. Conclusions

In this paper, we have presented a general consistent numerical methodology able to take into account strain rate dependence, inertial and thermal effects in structural computations for both metallic and composite material models. A thermomechanical implicit approach for element erosion to model material failure and crack propagation was also presented. The validity of the numerical model has been illustrated by different applications from the quasi-static regime to the dynamic one. All these coupled physical phenomena which are included in an implicit dynamic object-oriented finite element code are clearly an original contribution.

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