Physics

VERTICAL COSMIC RAY MUON FLUX AND CHARGE RATIO IN THE MOMENTUM REGION (0.3 GeV/c–2 GeV/c)

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The vertical cosmic ray muon flux and charge ratio have been determined at Chandigarh (λ = 21°N, altitude 661 m) in the momentum region (0.3 GeV/c–2 GeV/c) using a spark chamber magnet spectrograph. Corrections have been applied for the measurement and scattering errors, and bias in selecting the events. The corrected spectrum resembles the spectra determined by other workers in the momentum region (<1 GeV/c). However, towards the MDM of the spectrograph, the present spectrum is steeper as compared with the spectrum determined by other workers. The agreement between the present spectrum and the theoretical spectrum calculated after Olbert (1954) is fairly good in the same momentum region. The integral charge-ratio is 1.10±0.06.

INTRODUCTION

The spectrum of cosmic ray muons and their charge ratio are important physical quantities from the point of view of understanding the nuclear processes in which their parents are produced. This information can be obtained by comparing the shape and the absolute value of the muon momentum spectrum calculated from the primary spectrum with the one measured with a muon spectrograph at ground level.

Many workers (Allkofer et al., 1968, 1971; Hayman & Wolfendale, 1962; Bhattacharya, 1970, 1976; and Fukui et al., 1955, 1957) have reported the vertical muon flux in the lower momentum region at different geographical locations. The theoretical work on the production and propagation of cosmic ray muons through the atmosphere has been done by (Olbert, 1954; Barrett et al., 1952; Pal & Peters, 1964; Jabs, 1972; and Badhwar et al., 1977).

In this paper, the author reports his findings on the cosmic ray muon flux and charge ratio in the momentum region (0.3 GeV/c–2 GeV/c). A wide gap spark chamber magnet spectrograph has been used for determining these quantities. The salient features of the spectrograph have been discussed in the next section. The present spectrum, after being normalized to the Allkofer (1968) spectrum at 1 GeV/c, has been compared with the results of (Allkofer et al., 1968, Hayman & Wolfendale, 1962; Bhattacharya, 1970; and Fukui et al., 1957), and the theoretical calculations based on the work of Olbert (1954).

EXPERIMENTAL

The diagram of the experimental arrangement is illustrated in Fig. 1. The experiment was performed under lead absorber of thickness 80 g-cm⁻² and concrete of thickness 20 g-cm⁻². This amount of absorber is sufficient to completely suppress
the proton component of secondary cosmic rays arriving at sea-level; the pion intensity is already negligible (0.3). According to the calculations by Nagel (1965), it is sufficient to thermalize the electrons to 0.1 GeV/c by putting 3 radiation lengths equivalent of lead absorber over the spectrograph. The trajectory of muons was observed in two wide gap spark chambers whose dimensions were 15cm x 15cm x 10cm. A gas mixture of 90% Ne + 10% He, procured from Air Products, USA, was filled in the chambers at atmospheric pressure. A pair of pan-cake type GM-counters, each of dimensions 5cm x 5cm x 1cm, defines the acceptance area of the spectrograph. The structural details of the GM-counters have been discussed in Jain (1978). Briefly, the anode consists of a series of stainless steel wire, 2mil in diameter, stretched in the central plane of a perspex frame forming the body of the counter. A 2mil thick aluminium foil, stretched over the frame served as the cathode. Such a design considerably reduced the scattering of muons; moreover, they were easy to construct in the workshop. At a distance of 45cm between them, an acceptance area of 0.31cm² ster and acceptance angle of ± 7° are obtained. An air gap electromagnet is used to deflect the muons. The line-integral of the magnetic field is 57 KG-cm and is enough to produce measurable deflections in the trajectory of the particles. The average variation of the field over the sensitive volume of the magnet is 4% and the fringe field is 2% of the main field. One GM-counter and one spark chamber are placed above the magnet and the other GM-counter and the spark chamber are placed below the magnet. The distance between the spark chambers is 120cm.

The coincidence pulse of the GM-counters is applied to a gate circuit whose width is kept at 4 sec. The gated pulse simultaneously triggers a hydrogen thyratron 4C35 after amplification by beam power amplifier EFP60, and the camera driving and fiducials illuminating circuit. The plate of the thyratron is kept at +5 kV so that a sharp negative pulse of 5 kV amplitude is obtained on trigger.
This pulse triggers the first gap of a 10-stage Marx generator. The capacitors of the Marx generator, each of 2000 pf capacity, were charged in parallel upto \( \pm 12 \text{ kV} \) through 220 K, 2 W carbon resistors. The output pulse of the thyratron connects all the capacitors in series. A pulse of approximately 120 kV amplitude is thus obtained. This pulse is applied to both the spark chambers simultaneously. Since it requires only 8 kV/cm for breakdown to occur in the gas-mixture used, the excess charge is by-passed by shunting the chambers by 1500 ohm resistances. The shunt resistance sharpens the high voltage pulse and, consequently, the quality of the track is improved. The tracks were recorded photographically on ORWO NP55, 35mm film. The camera was placed at a distance of 3 meters from the centre of the magnet so that the entire spectrograph could be viewed directly. The fiducials were constructed by milling two horizontal and two vertical grooves in a perspex plate which were illuminated by four 9 V bulbs. One such assembly was placed in front of each spark chamber.

The GM-counters were filled with helium and ethyl alcohol vapour mixture at atmospheric pressure. The gas mixture was allowed to flow through the counters continuously so that their high efficiency could be maintained throughout the experiment.

With 0.31 cm\(^2\)-ster as the angle of acceptance and 7.3 \( \times 10^{-3} \text{ cm}^{-2}\text{sec}^{-1}\text{ster}^{-1} \) as the integral flux of muons (Allkofer & Jokisch, 1973), the expected count rate of the spectrograph is 9/hr. The observed average count rate is 7.8/hr. The discrepancy is probably due to finite delay in the coincidence circuit assembly and less than 100% efficiency of the GM-counters. The chance coincidence rate was 1%.

The data were analysed and measurements were made with a Mangia-spago meter at TIFR, Bombay.

RESULTS AND DISCUSSION

The experiment was run for 387hrs. At the time of analysis of the data, the following selection criteria were followed:—

(i) There should be only one track in each of the chambers; the number of multiple tracks was \( \sim 1\% \).

(ii) Each chamber should have a track.

(iii) The geometrical lay-out of the spectrograph was laid on the projection table of the mangia-spago meter and the tracks were projected on to it. If, on extrapolation, the tracks met within the magnetic field region, the event was accepted, otherwise rejected.

Following these criteria, events were selected and the angle of deflection and hence momentum for each event, was measured. The momentum spectrum so obtained was corrected by applying the corrections due to (a) measurement and scattering errors, (b) selection criterion (iii), and (c) absorber placed over the spectrograph.

The measurement and scattering errors were assumed to be Gaussian with the standard deviation:

\[
\sigma = \sqrt{(\sigma_{sc}^2 + \sigma_{meas})^2}
\]
If the deflection spectrum is given by
\[ \frac{dN}{d\phi} = f(\phi), \]
then the corrected spectrum takes the form
\[ \frac{dN}{d\phi} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} \exp \left[ -\frac{(\phi - \phi')^2}{2\sigma^2} \right] f(\phi') d\phi'. \] ..(1)

where \( \sigma \) is the angular resolution of the spectrograph and has the value 0.0095 radian for the present set-up.

Substituting \( \frac{(\phi - \phi')}{\sqrt{2\sigma}} = x, \)
eqn. (1) reduces to the following form
\[ \frac{dN}{d\phi} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} f(\phi - \sqrt{2\sigma}x) dx \] ..(2)

This expression was evaluated with the help of standard Gaussian quadrature tables for the following formula:
\[ \int_{-\infty}^{\infty} e^{-x^2} f(x) dx \approx \sum_{i=1}^{N} A_i f(x_i), \] ..(3)

where \( A_i \) are the appropriate weights.

Since \( p \alpha \frac{1}{\phi} \), the momentum spectrum could easily be obtained with the help of eqns. (2) and (3).

A monte-carlo simulation program was run for estimating the correction due to the selection criterion \((iii)\). A large number of muons were allowed to impinge normally on the upper chamber and they were followed in the lower chamber for different values of deflection of the particle. If, for a particular deflection, the tracks in the two chambers met within the magnetic field region, the event was recorded as 'success', otherwise a 'failure'. The process was repeated for different values of deflection.

The correction due to absorber placed above the spectrograph amounts to 200 MeV/c (Lawrence Radiation Laboratory Range-momentum tables).

The differential intensities of muons measured with the present spectrograph are shown in Table I, and Fig. (2).
### Table I

<table>
<thead>
<tr>
<th>Momentum interval (GeV/c)</th>
<th>Mean Mom. (GeV/c)</th>
<th>No. of +ve Muons</th>
<th>No. of —ve Muons</th>
<th>Differential intensity $10^{-9} [\text{cm}^{-2} \text{ sec}^{-1} \text{ sterd}^{-1} \text{ (GeV/c)}^{-1}]$</th>
<th>Charge-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3–0.4</td>
<td>0.35</td>
<td>59</td>
<td>61</td>
<td>2.68 ± 0.25</td>
<td>0.97 ± 0.17</td>
</tr>
<tr>
<td>0.4–0.5</td>
<td>0.45</td>
<td>64</td>
<td>57</td>
<td>2.68 ± 0.25</td>
<td>1.12 ± 0.20</td>
</tr>
<tr>
<td>0.5–0.7</td>
<td>0.6</td>
<td>117</td>
<td>108</td>
<td>2.60 ± 0.17</td>
<td>1.08 ± 0.14</td>
</tr>
<tr>
<td>0.7–1.0</td>
<td>0.85</td>
<td>136</td>
<td>121</td>
<td>2.30 ± 0.12</td>
<td>1.12 ± 0.14</td>
</tr>
<tr>
<td>1.0–1.4</td>
<td>1.2</td>
<td>124</td>
<td>114</td>
<td>1.75 ± 0.08</td>
<td>1.09 ± 0.14</td>
</tr>
<tr>
<td>1.4–1.7</td>
<td>1.55</td>
<td>64</td>
<td>60</td>
<td>1.20 ± 0.10</td>
<td>1.06 ± 0.19</td>
</tr>
<tr>
<td>1.7–2.2</td>
<td>1.95</td>
<td>85</td>
<td>67</td>
<td>0.9 ± 0.06</td>
<td>1.27 ± 0.2</td>
</tr>
</tbody>
</table>

**Fig. 2.** Differential spectrum of vertical muons. Experimental points (-); Form fit (— - -).
The experimentally observed spectrum has been fitted to the following empirical power law:

\[ N(p)dp = A p^{B+C} dp \]

where, \( A = 0.201 \times 10^{-2} \), \( B = -0.787 \), \( C = -0.502 \).

The agreement between the fitted spectrum and the observed one is very good except beyond 1.2 GeV/c, where the fitted spectrum becomes slightly steeper.

The present spectrum is normalized to the data of Allkofer (1968) at 1 GeV/c because the cut-off rigidities for these two stations are almost identical. The normalized spectrum is shown in Fig. (3). The spectra measured by other workers (Allkofer et al., 1968; Hayman & Wolfendale, 1962; Bhattacharya, 1970; and Fukui et al., 1957) are also shown in the same figure.

The differential spectrum of sea-level muons at Chandigarh (\( \lambda = 21^\circ \text{N} \)) has been derived from the production spectrum of Olbert (1954):

\[ \text{DIFFERENTIAL INTENSITY} \quad [\text{cm}^{-2} \text{s}^{-1} \text{stere}^{-1} \text{(GeV/c)}] \]

\[ \text{MOMENTUM} \quad [\text{GeV/c}] \]

**Fig. 3.** Comparison of the present spectrum with the spectrum of other workers.
\( G(R', \lambda) = \frac{7.34 \times 10^4}{[a(\lambda) + R]^{3.58}} \, g^{-2} \text{cm}^2 \text{sec}^{-1} \text{sterd}^{-1}, \)

where \( a(\lambda) \) is the latitude parameter which is 625 g-cm\(^{-2} \) for \( = 21^\circ \text{N} \), by using the relation

\[
I(d, R; \lambda) = \int_0^d G(R+d-x; \lambda) \exp(-x/L) \, w(x, R) \, dx,
\]

where \( R' = R + d - x; w(x, R) \) is the survival probability of a muon produced at the atmospheric depth \( x \) reaching a depth \( d = 1030 \, \text{g-cm}^{-2} \) with residual range \( R; L \), the mean attenuation length, is 120 g-cm\(^{-2} \). The values of survival probabilities were taken from Olbert (1953). The calculated spectrum is shown in Fig. (3).

It is seen from Fig. (3) that the spectral shape of the present spectrum agrees well with the spectra measured by Allkofer (1968), Fukui (1957), Bhattacharya (1970) upto \( \sim 1.2 \, \text{GeV/c} \). If the average statistical fluctuations of 10 per cent are taken into consideration, then the normalized values of the present spectrum agree well with the absolute values of Allkofer \textit{et al.} (1968). The present spectrum lies slightly above the other spectra discussed above. This can be ascribed to the latitude effect on muon spectrum in this momentum region. There is, however, a considerable difference between the Durham spectrum (Hayman & Wolfendale, 1962) and the present spectrum. The possible sources of disagreement are: \( (i) \) geomagnetic effect; \( (ii) \) effect caused by the normal variation of the atmospheric temperature with latitude and the influence of the atmospheric density on the decay of meson; \( (iii) \) multiple coulomb scattering in the detector materials and absorber; \( (iv) \) bias effects in rejection of accompanying muon events; and \( (v) \) solar activity. The higher intensities of the Durham spectrum can be explained by the latitude effect only upto a momentum of \( \sim 0.9 \, \text{GeV/c} \). No definite conclusions can be drawn about the effect of the solar activity because proper information is not available. The bias effects are also not important in the momentum region under consideration.

Beyond 1.2 \, \text{GeV/c}, the present spectrum is steeper as compared with the other spectra. In the momentum region beyond 1.5 \, \text{GeV/c}, the spectra due to Allkofer \textit{et al.}, (1968) and Bhattacharya (1970) are much flatter. The difference is large because this momentum region lies near the maximum-detectable-momentum of the present spectrograph. Consequently, the measurement errors would become dominant. Gaussian errors were assumed for the measurement of the angle of deflection of muon in the magnetic field, the distortion in the spectrum of angle of deflection due to these errors was calculated, and then corrections were applied to the measured spectrum. These calculations are feasible and correct in principle only if the error distributions of the measurements are known precisely. Because of the steepness of the spectrum beyond 1 \, \text{GeV/c}, a small deviation from a Gaussian error distribution in the measurement of the angle of deflection could generate relatively large errors in the momentum spectra.
A comparison of the present spectrum with the spectrum calculated after Olbert (1954) shows that there is a reasonably good agreement between the general shapes of the two spectra in the momentum region $<1.2$ GeV/c. However, the calculated intensities in this region are considerably lower than the measured intensities.

**MUON CHARGE-RATIO**

In Fig. (4) the measured charge-ratio of muon is shown as a function of momentum. The integral charge-ratio is $1.10 \pm 0.06$. The measurements of other workers (Allkofer et al., 1968; Fukui et al., 1955, 1957) have also been shown in this figure. These measurements were done at different geographical locations. But this quantity is largely insensitive to the latitude effect. It is seen from the figure that in the momentum region under investigation, values from 0.85 to 1.65 have been reported. The difference between the present value and the values reported by others can possibly be attributed to a finite contamination by slower and knock-on electrons. However, it is difficult to get electrons of momentum $>0.3$ GeV/c (the region under investigation).

The $\mu^+ / \mu^-$ ratio reflects the composition of primary cosmic rays. A lower value of this quantity would mean that the proportion of protons in the primary cosmic rays is smaller than the presently accepted value. But more elaborate experiments are needed to verify such a conjecture.

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