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PSEUDO-SLANT SUBMANIFOLDS OF A SASAKIAN MANIFOLD

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Carriazo [2] defined the notion of bi-slant submanifolds of an almost Hermitian manifold. As a special case of these submanifolds, he introduced anti-slant submanifolds which he, later called as pseudo-slant submanifolds [2]. The purpose of the present paper is to study the pseudo-slant submanifolds of a Sasakian manifold. In this paper we work out integrability conditions of distributions on these submanifolds and also, obtain a few interesting results of this setting.

Key Words: Sasakian Manifold; Slant Submanifold; Pseudo-Slant Submanifold

1. INTRODUCTION

The geometry of slant immersions was initiated by Chen [4] as a natural generalization of both holomorphic and totally real immersions. Many authors have studied slant immersions in almost Hermitian manifolds. Lotta [3] introduced the notion of slant immersions in contact manifolds. Cabrerizo *et al*. [6] studied and characterized slant submanifolds of K-contact and Sasakian manifolds and have given several examples of such immersions. Recently, Carriazo [2] defined and studied bi-slant immersions in almost Hermitian manifolds and simultaneously gave the notion of pseudo-slant submanifold in almost Hermitian manifolds. The purpose of the present paper is to define and study the contact version of pseudo-slant submanifolds. In section 2 we review and collect some necessary results. In section 3 we define pseudo-slant submanifolds of contact manifolds. In particular, we study the pseudo-slant submanifolds in the setting of Sasakian manifolds and work out integrability conditions of distributions involved in the definition of pseudo-slant submanifolds and have also obtained some geometrically significant results of this setting.

2. PRELIMINARIES

Let $(\bar{M}, g)$ be an odd dimensional Riemannian manifold, $T\bar{M}$ the Lie algebra of vector fields in $\bar{M}$. Then $\bar{M}$ is said to be an almost contact metric manifold [5], if there exist on $\bar{M}$ a tensor field ϕ of type (1, 1) and a global vector field ξ (known as structure vector field) such that, if η is the dual 1-form of ξ then

$$\begin{aligned} \phi^2 X &= -X + \eta(X)\xi, \qquad g(X,\xi) = \eta(X) \\ g(\phi X, \phi Y) &= g(X,Y) - \eta(X)\eta(Y). \end{aligned}$$

From the above relations, it also follows that

$$g(\phi X, Y) = -g(X, \phi Y) \tag{1}$$

for any $X, Y \in T\bar{M}$. Let Φ denote a 2-form in $\bar{M}$ given by $\Phi(X,Y) = g(X,\phi Y)$ for all $X, Y \in T\bar{M}$. The 2-form Φ is then called the fundamental 2-form on $\bar{M}$. The manifold $\bar{M}$ is said to be a contact metric manifold if $\Phi = d\eta$.

If ξ is a Killing vector field with respect to g, the contact metric structure is called K-contact structure. It is known that a contact metric manifold is K-contact if and only if $\bar{\nabla}_X \xi = -\phi X$, for any $X \in T\bar{M}$, where $\bar{\nabla}$ denotes the Levi-Civita connection on $\bar{M}$. The almost contact structure of $\bar{M}$ is said to be normal if $[\phi,\phi] + 2d\eta \otimes \xi = 0$, where $[\phi,\phi]$ is the Nijenhuis tensor of ϕ. A Sasakian manifold is a normal contact metric manifold. Every Sasakian manifold is a K-contact manifold. It is known that an almost contact metric manifold is a Sasakian manifold if and only if

$$(\bar{\nabla}_X \phi)Y = g(X,Y)\xi - \eta(Y)X. \tag{2}$$

Moreover, on a Sasakian manifold $\bar{M}$

$$\bar{\nabla}_X \xi = -\phi X \tag{3}$$

for any $X, Y \in T\bar{M}$ and ξ is the structure vector field.

Now, let M be a submanifold immersed in $\bar{M}$; we denote by the same symbol g the induced metric on M. Let TM be the Lie-algebra of vector fields on M and $T^{\perp}M$ the set of all vector fields normal to M. Then the Gauss and Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{4}$$

$$\bar{\nabla}_X V = -A_V X + \nabla^{\perp}_X V \tag{5}$$

for any $X, Y \in TM$ and $V \in T^{\perp}M$, where $\nabla^{\perp}$ is the connection in the normal bundle, h is the second fundamental form of M and A_V is the shape operator associated with V. The second fundamental form h and the shape operator A_V are related by

$$g(A_V X, Y) = g(h(X,Y), V). \tag{6}$$

For any $X \in TM$ and $V \in T^{\perp}M$, we write

$$\phi X = TX + NX \tag{7}$$

$$\phi V = tV + nV \tag{8}$$

where TX (resp. tV) denotes the tangential part of ϕX (resp. ϕV) and NX (rsep. nV) denotes the normal part of ϕX (resp. ϕV). The covariant derivative of T and N are defined as

$$(\bar{\nabla}_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{9}$$

$$(\bar{\nabla}_X N)Y = \nabla^{\perp}_X NY - N\nabla_X Y. \tag{10}$$

The submanifold M is invariant if N is identically zero, that is , $\phi X \in TM$, for any $X \in TM$. On the other hand, M is an anti-invariant submanifold if T is identically zero, that is, $\phi X \in T^{\perp}M$, for any $X \in TM$. The distribution spanned by the structure vector field ξ is denoted by $\langle\xi\rangle$. By applying equation (4) and (7) on (3), it follows that

$$(a)\ \nabla_X \xi = -TX \quad \text{and} \quad (b)\ h(X, \xi) = -NX \tag{11}$$

for all $X \in TM$.

3. Pseudo-Slant Submanifolds of Almost Hermitian and Almost Contact Metric Manifolds

In this section we study pseudo-slant submanifolds of Sasakian manifold and obtain integrability conditions of the distributions on pseudo-slant submanifolds. To begin with, we show how to obtain pseudo-slant submanifolds of almost Hermitian manifolds by slant submanifold of contact manifolds.

Let M be a Riemannian manifold, isometrically immersed in an almost contact metric manifold $(\bar{M}, \phi, \xi, \eta, g)$. From now on, we suppose that the structure vector field ξ is tangent to M. Hence, if we denote by D the orthogonal distribution to ξ in TM, then

$$TM = D \oplus \langle\xi\rangle.$$

In this setting, for each nonzero vector X tangent to M at x, such that X is not proportional to ξ_x, we denote by $\theta(X)$ the angle between ϕX and D_x. Then, M is said to be slant [3] if the angle $\theta(X)$ is constant, which is independent of the choice of $x \in M$ and $X \in T_xM - \langle\xi_x\rangle$. The angle θ of the slant immersion is called the slant angle of the immersion. Invariant and anti-invariant immersions are slant immersions with slant angle $\theta = 0$ and $\theta = \pi/2$, respectively. A slant immersion which is neither invariant nor anti-invariant is called a proper slant immersion. In the setting of submanifolds of an almost Hermitian manifolds, slant distribution are defined on the same lines. They are defined as follows.

Given a submanifold S, isometrically immersed in an almost Hermitian manifold $(\bar{S}, J, g_1)$, a differentiable distribution ν on S is said to be a slant distribution, if for any nonzero $X \in \nu_x$, $x \in S$, the angle between JX and the vector space ν_x is constant, that is, it is independent of choice of $x \in S$ and $X \in \nu_x$. This constant angle is called the slant angle of the slant distribution ν.

The following theorem provides a useful characterization for the existence of a slant distribution on a contact metric manifold.

***Theorem* 1** [7] — Let ν be a distribution on M, orthogonal to ξ. Then, ν is slant if and only if there exists a constant $\lambda \in [0, 1]$ such that $(PT)^2X = -\lambda X$, for any $X \in \nu$, where P denotes the orthogonal projection on ν. Furthermore, in this case $\lambda = \cos^2 \theta_\nu$.

A submanifold S of an almost Hermitian manifolds $\bar{S}$ is said to be pseudo-slant submanifold if there exist on S two orthogonal distributions D_1 and D_2 such that $TS = D_1 \oplus D_2$, where D_1 is totally real distribution (i.e., $JD_1 \subset T^\perp S$) and D_2 is slant distribution with the slant angle $\theta \neq \pi/2$ (see [2]). In particular, if dim $D_1 = 0$ and $\theta \in (0, \pi/2)$, then S is a proper slant submanifold of almost Hermitian manifold $\bar{S}$ introduced by Chen [4].

In the following paragraphs, we show that there is a relationship between slant submanifolds of almost contact metric manifolds and pseudo-slant submanifolds of almost Hermitian manifolds.

Let $(\bar{M}, \phi, \xi, \eta, g)$ be an almost contact metric manifold. Then we consider the manifold $\bar{M} \times R$ and denote by $(X, f\frac{d}{dt})$ a vector field of $\bar{M} \times R$, where X is tangent to $\bar{M}$, t is the coordinate of R and f is a differentiable function on $\bar{M} \times R$. An almost complex structure J on this manifold is defined as

$$J\left(X, f\frac{d}{dt}\right) = \left(\phi X - f\xi, \eta(X)\frac{d}{dt}\right). \tag{12}$$

It is well known that $(\bar{M} \times R, J, g_1)$ is an almost Hermitian manifold [9], where g_1 denotes the product metric given by

$$g_1\left(\left(X, f\frac{d}{dt}\right), \left(Y, h\frac{d}{dt}\right)\right) = g(X, Y) + fh.$$

We now recall the following important result due to Lotta.

***Theorem* 2** [3] — Let M be a slant submanifold of dimension n of an almost contact metric manifold $\bar{M}$ with slant angle $\theta \neq \pi/2$. Then we have

$$n \text{ is even } \Leftrightarrow \xi \text{ is orthogonal to } M$$
$$n \text{ is odd } \Leftrightarrow \xi \text{ is tangent to } M.$$

The following result provides a method to obtain a pseudo-slant submanifold of $\bar{M} \times R$ from slant submanifold of $\bar{M}$.

***Theorem* 3** — Let M be a non anti-invariant even dimensional slant submanifold of an almost contact metric manifold $\bar{M}$. Then $M \times R$ is a pseudo-slant submanifold of the almost Hermitian manifold $\bar{M} \times R$, with totally real distribution $D_1 = \left\{\left(0, \frac{d}{dt}\right)\right\}$ and slant distribution

$D_2 = \{(X, 0)|X \in D\}$.

PROOF : It is clear that distribution D_1 and D_2 are orthogonal and $T(M \times R) = D_1 \oplus D_2$. Moreover, by virtue of equation (12)

$$J\left(0, \frac{d}{dt}\right) = -(\xi, 0).$$

Hence, D_1 is a totally real distribution in view of Theorem 2. Finally, it is easy to see that D_2 is a slant distribution in the sense of Papaghiuc [9].

To introduce pseudo-slant submanifold of an almost contact metric manifold; first we recall the definition of Bi-slant submanifold.

Definition 1[7] — M is said to be a Bi-slant submanifold of an almost contact metric manifold $\bar{M}$ if there exist two orthogonal distributions D_1 and D_2 on M such that

(i) TM admits the orthogonal direct decomposition $TM = D_1 \oplus D_2 \oplus \langle\xi\rangle$

(ii) The distribution D_1 is slant with angle θ_1

(iii) The distribution D_2 is slant with angle θ_2

Definition 2 — We say that M is a pseudo-slant submanifold of an almost contact metric manifold $\bar{M}$ if there exist two orthogonal distributions D_1 and D_2 on M such that

(i) TM admits the orthogonal direct decomposition $TM = D_1 \oplus D_2 \oplus \langle\xi\rangle$

(ii) The distribution D_1 is anti-invariant i.e., $\phi D_1 \subseteq T^{\perp}M$

(iii) The distribution D_2 is slant with slant angle $\theta \neq \pi/2$

From the above definition it is clear that if $\theta = 0$, then the pseudo-slant submanifold is a semi-invariant submanifold. On the other hand if we denote the dimension of D_i by d_i, for $i = 1, 2$, then we find the following cases

(a) If $d_2 = 0$ then M is an anti-invariant submanifold.

(b) If $d_1 = 0$ and $\theta = 0$, then M is an invariant submanifold.

(c) If $d_1 = 0$ and $\theta \neq 0$, then M is a proper slant submanifold, with slant angle θ.

A pseudo-slant submanifold is proper if $d_1 d_2 \neq 0$ and $\theta \neq 0$.

If we put $\theta_1 = \pi/2$ and $\theta_2 = \theta \in [0, \pi/2)$ in the following example of 5-dimensional bi-slant submanifold M with slant angles θ_1 and θ_2 (see [7])

$$x(u, v, w, s, t) = 2(u, 0, w, 0, v\cos\theta_1, v\sin\theta_1, s\cos\theta_2, s\sin\theta_2, t) \tag{13}$$

then it becomes a pseudo-slant submanifold in R [9]. Furthermore, it is easy to see that

$$\begin{aligned} e_1 &= 2\left(\frac{\partial}{\partial x^1} + y^1\frac{\partial}{\partial Z}\right), \quad e_2 = 2\frac{\partial}{\partial y^2} \quad e_3 = 2\left(\frac{\partial}{\partial x^3} + y^3\frac{\partial}{\partial Z}\right) \\ e_4 &= 2\cos\theta\frac{\partial}{\partial y^3} + 2\sin\theta\frac{\partial}{\partial y^4} \quad e_5 = 2\frac{\partial}{\partial Z} = \xi \end{aligned}$$

form a local orthonormal frame of TM. We define, the distributions $D_1 = \langle e_1, e_2\rangle$ and $D_2 = \langle e_3, e_4\rangle$. Then it is clear that $TM = D_1 \oplus D_2 \oplus \langle\xi\rangle$ in which D_1 is anti-invariant distribution and D_2 is slant distribution with slant angle θ.

Suppose M to be a pseudo-slant submanifold of an almost contact metric manifold $\bar{M}$. Then, for any $X \in TM$, put

$$X = P_1X + P_2X + \eta(X)\xi \tag{14}$$

where P_i $(i = 1, 2)$ are projection maps on the distributions D_1 and D_2. Now operating ϕ on both sides of equation (14)

$$\phi X = NP_1X + TP_2X + NP_2X. \tag{15}$$

It is easy to see that

$$TX = TP_2X, \quad NX = NP_1X + NP_2X \tag{16}$$

and,

$$\phi P_1X = NP_1X, \quad TP_1X = 0; \tag{17}$$

$$TP_2X \in D_2. \tag{18}$$

Since D_2 is slant distribution, by Theorem 1

$$T^2X = -\cos^2\theta X \tag{19}$$

for any $X \in D_2$.

Now, we have the following theorem:

***Theorem* 4** — Let M be a submanifold of an almost contact metric manifold $\bar{M}$, such that $\xi \in TM$. Then M is a pseudo-slant submanifold if and only if there exists a constant $\lambda \in (0, 1]$ such that

(i) $D = \{X \in TM \mid T^2X = -\lambda X\}$ is a distribution on M

(ii) For any $X \in TM$, orthogonal to D, $TX = 0$.

Furthermore, in this case $\lambda = \cos^2\theta$, where θ denotes the slant angle of D.

PROOF: Set $\lambda = \cos^2\theta$, then it follows from (17) and (18) that $D = D_2$. Conversely, consider the orthogonal direct decomposition $TM = D \oplus D^\perp \oplus \langle\xi\rangle$. It is evident that $TD \subseteq D$. Hence, by using statement

(ii) it is clear that $D^\perp$ is an anti-invariant distribution. Finally, Theorem 1 and statement

(i) imply that D is a slant distribution, with slant angle θ satisfying $\lambda = \cos^2\theta$.

Now, we will discuss the integrability of distributions involved in a pseudo-slant submanifold of a Sasakian manifold.

If μ is the invariant subspace of the normal bundle $T^\perp M$ then, in the case of pseudo-slant submanifold, the normal bundle $T^\perp M$ can be decomposed as follows

$$T^\perp M = \mu \oplus ND_1 \oplus ND_2. \tag{20}$$

As D_1 and D_2 are orthogonal distributions on M, $g(Z,X) = 0$ for each $Z \in D_1$ and $X \in D_2$. Thus, by equation (7) and (1), we may write

$$g(NZ, NX) = g(\phi Z, \phi X) = g(Z, X) = 0.$$

That means the distributions ND_1 and ND_2 are mutually perpendicular. Infact, the decomposition (20) is an orthogonal direct decomposition.

For a pseudo-slant submanifold of a Sasakian manifold the following lemmas play an important role in working out the integrability conditions of the distributions involved in this setting.

Lemma 1 — Let M be a pseudo-slant submanifold of a Sasakian manifold $\bar{M}$. Then

$$A_{\phi Y}X = A_{\phi X}Y \tag{21}$$

for all $X, Y \in D_1$.

PROOF: For any X, Y in D_1 and Z in TM, using (6), (1), (3) and (4) we find that

$$\begin{aligned} g(A_{\phi Y}X, Z) &= -g(\phi\bar{\nabla}_Z X - \phi\nabla_Z X, Y) \\ &= -g(\phi\bar{\nabla}_Z X, Y) \end{aligned}$$

i.e.,

$$g(A_{\phi Y}X, Z) = -g(\bar{\nabla}_Z \phi X - (\bar{\nabla}_Z \phi)X, Y).$$

On applying equations (2) and (5) the above equation yields

$$g(A_{\phi Y}X, Z) = g(A_{\phi X}Y, Z)$$

The result follows from the above equation.

Lemma 2 — Let M be a pseudo-slant submanifold of a Sasakian manifold $\bar{M}$, then

$$[X, \xi] \in D_1 \tag{22}$$

for all $X \in D_1$.

PROOF: For any $X \in D_1$ and $Z \in D_2$

$$g([X, \xi], TZ) = g(\bar{\nabla}_\xi TZ, X). \tag{23}$$

On using equations (2) $\&$ (7) we have

$$\bar{\nabla}_\xi TZ = -\bar{\nabla}_\xi NZ + \phi\bar{\nabla}_\xi Z.$$

Applying the above formula in (23) and using (6), (7), (4) (5) and $(11)(b)$, we get

$$g([X, \xi], TZ) = -g(NX, NZ) + g(NX, NZ) = 0.$$

This proves the lemma completely.

Lemma 3 — Let M be a pseudo-slant submanifold of a Sasakian manifold $\bar{M}$ then, for any $X, Y \in D_1 \oplus D_2$

$$g([X,Y],\xi) = 2g(X,TY). \tag{24}$$

The proof of equation (24) is straightforward and may be obtained by using $(11)(a)$.

Proposition 1 — Let M be a pseudo-slant submanifold of a Sasakian manifold $\bar{M}$. Then, anti-invariant distribution D_1 is integrable.

PROOF: For any $X, Y \in D_1$ and $Z \in D_2$, by (14)

$$g([X,Y],TP_2Z) = -g(\phi[X,Y],P_2Z)$$

Now, using (2) and (5), we find

$$g([X,Y],TP_2Z) = g(A_{\phi Y}X - A_{\phi X}Y, P_2Z).$$

Now, the integrability of the distribution D_1 follows on using equations (21) and (24).

Corollary 1 — On a psedo-slant submanifold M of a Sasakian manifold $\bar{M}$, the distribution $D_1 \oplus \langle\xi\rangle$ is integrable.

The corollary follows from Proposition (1) and equation (22).

Lemma 4 — Let M be a pseudo-slant submanifold of a Sasakian manifold $\bar{M}$. Then, the slant distribution D_2 is not integrable.

By the definition of pseudo-slant submanifold and in view of equation (24) the result follows.

Proposition 2 — Let M be a pseudo-slant submanifold of $\bar{M}$ then the distribution $D_2 \oplus \langle\xi\rangle$ is integrable if and only if

$$h(Z,TW) - h(W,TZ) + \nabla^{\perp}_{Z}NW - \nabla^{\perp}_{W}NZ$$

lies in ND_2 for each $Z, W \in D_2 \oplus \langle\xi\rangle$.

PROOF : Making use of equations (7), (2), (4) and (5), we obtain

$$g(N[Z,W],NX) = g(h(Z,TW) - h(W,TZ) + \nabla^{\perp}_{Z}NW - \nabla^{\perp}_{W}NZ, NX)$$

for each $X \in D_1$ and $Z, W \in D_2$. The result follows on using the fact that ND_1 and ND_2 are mutually perpendicular.

Cabrerizo *et al*. [6] have obtained the following expression for ∇T in case of some slant submanifolds of Sasakian manifolds

$$(\nabla_X T)Y = \cos^2\theta(g(X,Y)\xi - \eta(Y)X) \tag{25}$$

for any $X, Y \in TM$.

Note — 1 Formula (25) provides a sufficient condition for a submanifold to be a proper slant submanifold of a K-contact manifold (see [6]).

Consider the 5-dimensional pseudo-slant submanifold of R^9 given by equation (13) i.e.,

$$x(u,v,w,s,t) = 2(u,0,w,0,0,v,s\cos\theta, s\sin\theta, t)$$

where $\theta \in (0,\pi/2)$ is the slant angle of the slant distribution D_2. Then it is easy to see that

$$(\nabla_X T)Y = \cos^2\theta(g(P_2X,Y)\xi - \eta(Y)P_2X) \tag{26}$$

for any $X, Y \in TM$. If we take $X = P_2X + \eta(X)\xi$ and $Y = P_2Y + \eta(Y)\xi$ then (26) implies that

$$(\nabla_X T)Y = \cos^2\theta(g(X,Y)\xi - \eta(Y)X)$$

Thus, in view of note (1), the distribution $D_2 \oplus \langle\xi\rangle$ works as the tangent bundle of proper slant submanifold. On the other hand if $X, Y \in D_1 \oplus \langle\xi\rangle$ then (26) implies

$$(\nabla_X T)Y = 0$$

which shows that anti-invariant submanifolds satisfy the above equation. We want to realize that (26) is a natural condition for pseudo-slant submanifolds of a Sasakian manifold analogous to slant and semi-slant submanifolds of Sasakian manifolds worked out by Cabrerizo *et al*. (see [6, 7]. Now, we have the following theorem

***Theorem* 5** — Let M be a proper pseudo-slant submanifold with angle θ, of a Sasakian manifold $\bar{M}$. Then, for any $X, Y \in TM$.

$$(\nabla_X T)Y = A_{NP_1Y}X + A_{NP_2Y}X + th(X,Y) + g(X,Y)\xi - \eta(Y)X. \tag{27}$$

Hence, M satisfies (26) if and only if

$$A_{NY}X = A_{NX}Y + \eta(Y)P_1X - \eta(X)P_1Y - \sin^2\theta(\eta(X)P_2Y - \eta(Y)P_2X) \tag{28}$$

where $NX = NP_1X + NP_2X$

PROOF : For any $X, Y \in TM$

$$\bar{\nabla}_X \phi Y = (\bar{\nabla}_X \phi)Y + \phi\bar{\nabla}_X Y$$

Now using (2), (7), (4), (5) and (9) and comparing tangential parts we obtain (27).

Suppose M is a proper pseudo-slant submanifold satisfying (26). Then, by (27)

$$\begin{aligned}\cos^2\theta(g(P_2X,Y)\xi - \eta(Y)P_2X) \quad &= \quad A_{NP_1Y}X + A_{NP_2Y}X + th(X,Y) + g(P_1X,Y)\xi \\ &\quad\quad + g(P_2X,Y)\xi - \eta(Y)P_1X - \eta(Y)P_2X\end{aligned}$$

or,

$$\begin{aligned} A_{NP_1Y}X + A_{NP_2Y}X &= -th(X,Y) - g(P_1X,Y)\xi + \eta(Y)P_1X \\ &\quad - \sin^2\theta(g(P_2X,Y)\xi - \eta(Y)P_2X). \end{aligned} \tag{29}$$

Similarly,

$$\begin{aligned} A_{NP_1X}Y + A_{NP_2X}Y &= -th(X,Y) - g(P_1Y,X)\xi + \eta(X)P_1Y \\ &\quad - \sin^2\theta(g(P_2Y,X)\xi - \eta(X)P_2Y), \end{aligned}$$

Finally, we have

$$A_{NY}X = A_{NX}Y + \eta(Y)P_1X - \eta(X)P_1Y - \sin^2\theta(\eta(X)P_2Y - \eta(Y)P_2X)$$

Conversely, suppose (28) holds. Then for any $Z \in TM$

$$\begin{aligned} g(A_{NY}X,Z) &= -g(th(Y,Z),X) + \eta(Y)g(P_1X,Z) - \eta(X)g(P_1Y,Z) \\ &\quad - \sin^2\theta(\eta(X)g(P_2Y,Z) - \eta(Y)g(P_2X,Z)). \end{aligned} \tag{30}$$

Interchanging X and Z, and making use of the fact that $g(P_1X,Y) = g(X,P_1Y)$, for each $X,Y \in TM$, we get

$$\begin{aligned} g(A_{NY}X,Z) &= -g(th(X,Y),Z) + \eta(Y)g(P_1X,Z) - \eta(Z)g(P_1X,Y) \\ &\quad - \sin^2\theta(\eta(Z)g(P_2X,Y) - \eta(Y)g(P_2X,Z)). \end{aligned}$$

Taking account of equation (14), the above equation yields

$$\begin{aligned} g(A_{NY}X,Z) &= -g(th(X,Y),Z) + \eta(Y)g(X,Z) - \eta(Z)g(X,Y) \\ &\quad + \cos^2\theta(\eta(Z)g(P_2X,Y) - \eta(Y)g(P_2X,Z)). \end{aligned}$$

Now, by using equation (27), we obtain

$$(\nabla_X T)Y = \cos^2\theta(g(P_2X,Y)\xi - \eta(Y)P_2X)$$

which proves the assertion.

***Theorem* 6** — Let M be a proper pseudo-slant submanifold of a Sasakian manifold $\bar{M}$ with slant angle θ, then

(i) M satisfies (26) if and only if

$$(\nabla_X TP_2)Y = \cos^2\theta(g(P_2X,Y)\xi - \eta(Y)P_2X)$$

(ii) If M satisfies (26), then

$$\nabla_X Z \in D_1 \oplus \langle\xi\rangle, \quad \nabla_X W \in D_2 \oplus \langle\xi\rangle$$

for any $X, Y \in TM$, $Z \in D_1$ and $W \in D_2$.

PROOF : For any pseudo-slant submanifold of Sasakian manifold $\bar{M}$,
$T = TP_2$. Then statement
(i) follows from (26) and (16).

Suppose that M satisfies (26). Then by statement (i)

$$(\nabla_X TP_2)Y = \cos^2\theta(g(P_2X, Y)\xi - \eta(Y)P_2X.$$

From above equation it is evident that

$$P_1\nabla_X TP_2 Y = 0 \text{ i.e,} \quad \nabla_X TP_2 Y \in D_2 \oplus \langle\xi\rangle$$

or, equivalently

$$\nabla_X Z \in D_2 \oplus \langle\xi\rangle \tag{31}$$

for any $X \in TM$ and $Z \in D_2$. Then (ii) follows from (30).

Now, we have the following corollary:

Corollary 2 — Let M be a pseudo-slant submanifold of a Sasakian manifold $\bar{M}$ such that M satisfies (26). Then $D_2 \oplus \langle\xi\rangle$ is integrable and leaves of distributions $D_1 \oplus \langle\xi\rangle$ and $D_2 \oplus \langle\xi\rangle$ are totally geodesic in M.

The corollary follows by statement (ii) of Theorem 6 and $(11)(a)$.

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