A FORMULA FOR $C(T)$ IN GUPTA'S PAPER

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§1. In the preceding paper*, Hansraj Gupta tells how I brought to his attention the problem which he discusses, and how we worked on it independently and solved it. In this short note, I say briefly what the problem was and what remained to be done about it.

Suppose that we have a circle whose circumference is divided into $n$ equal arcs, the points of division being marked by dots. Any $k$ of these points may be joined by straight lines to form a convex $k$-gon. The arc-lengths of the sides of this $k$-gon sum up to $n$. If rotations and reflections are considered to be redundant, Gupta finds in his paper, an enumeration function $R(n, k)$ giving the number of possible $k$-gons, different from each other in the sense that none of these can be obtained from any other by rotation or/and reflection. In finding the formula for $R(n, k)$, use was made of functions $C(T)$ which represent the contribution to $R(n, k)$ of any partition of $n$ of the type

$$T = (t_1, t_2, \ldots, t_i), \quad t_1 + t_2 + \ldots + t_i = k$$

into $k$ parts, i.e. one which has $t_1$ parts each equal to $b_1$; $t_2$ parts each equal to $b_2$; \ldots; and $t_i$ parts each equal to $b_i$. Without loss of generality, we may take

$$t_1 \leq t_2 \leq \ldots \leq t_i.$$

Recall that $C(T)$ does not depend on the size of the $b$'s (all distinct of course) but only on their frequencies i.e. $t$'s.

Beyond giving a few 'easy to prove' rules (see section 3.3 of Gupta's paper), which do not cover all the cases that arise, Gupta does not say anything about the evaluation of $C(T)$ for any given $T$.

The object of this note is to give a general formula for $C(T)$ which will be as pretty as Gupta's for $R(n, k)$. While I am certain that the formula is correct, I must leave the proof to the reader or to Professor Gupta, for I simply do not know how to do it, my excuse being that I am not really a mathematician.

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*See pages 964-999 of this issue of the Journal.
§2. _The formula for $C(T)$_. — Let $\text{g.c.d.} (t_1, t_2, \ldots, t_i) = g$, then we have

$$2C(T) = S(T) + C'(T),$$

where

$$S(T) = u \frac{(\lfloor t_1/2 \rfloor + \lfloor t_2/2 \rfloor + \ldots + \lfloor t_i/2 \rfloor)!}{[t_1/2]! \ [t_2/2]! \ \ldots \ [t_i/2]!}$$

with

$$u = 0 \text{ if at least three of the } t \text{'s are odd;}$$

$$= 1 \text{ otherwise;}$$

and

$$C'(T) = \frac{1}{k} \sum_{d \mid k} \phi(d) \frac{(k/d)!}{(t_1/d)! \ (t_2/d)! \ \ldots \ (t_i/d)!}.$$

As the reader will readily guess, my cue came from the rules given by Gupta in his paper and the formula

$$2R(n, k) = S(n, k) + \frac{1}{k} \sum_{d \mid (n, k)} \phi(d) \left( \frac{n}{d} - 1; \frac{k}{d} - 1 \right).$$

**Examples**

(i) \[ C'(6, 6) = \frac{1}{12} \left\{ \frac{12!}{6! \ 6!} + \frac{6!}{3! \ 3!} + 2 \cdot \frac{4!}{2! \ 2!} + 2 \cdot \frac{2!}{1! \ 1!} \right\} \]

\[= \frac{(924 + 20 + 12 + 4)}{12} = 80; \]

and \[ S(6, 6) = \frac{6!}{3! \ 3!} = 20. \]

Hence \[ C(6, 6) = 50. \]

(ii) \[ C'(3, 3, 3) = \frac{1}{9} \left\{ \frac{9!}{3! \ 3! \ 3!} + 2 \cdot \frac{3!}{1! \ 1! \ 1!} \right\} \]

\[= \frac{(1680 + 12)}{9} = 188; \]

and \[ S(3, 3, 3) = 0; \]

so that \[ C(3, 3, 3) = 94. \]