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ĀRYABHAṬA AND THE TABLE OF RSINES

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It is argued that there is no real evidence for the claim that Āryabhaṭa derived his table of Rsines from a table of chords of Hipparchus, contrary to the assertions of Toomer.

Key words: Āryabhaṭa, Table of RSines

INTRODUCTION

In a recent paper, Abhayankar[1] has argued that "Āryabhaṭa's values of *bhagaṇas* were probably derived from the Babylonian Planetary data." Kak[2] has refuted this theory ofAbhyankar and has shown that there is no basis for such assertion and that infact, Āryabhaṭa created a very original and novel siddhānta. Another theory[3-6] that floats around is that Āryabhaṭa's Table of RSines[7] was derived from the Table of Chords of Hipparchus. Three of the authors of the four cited above, namely, Neugebauer[3], Van der Waerden[4], and Pingree[5], all renowned historians of astronomy, base their conclusions on the authority of the fourth, Toomer[6]. They proclaim that Toomer has proved beyond doubt that Āryabhaṭa's Table of RSines was derived from the Table of Chords of Hipparchus. How does Toomer prove this "fact" or has he really proved it?

On a closer examination of Toomer's paper (as has been done by Thomson[8]), it is found that Toomer bases his work on the suggestion of none other than Neugebauer himself! There are no surviving documents containing the Hipparchus Chord Table, not even in a fragmentary form. Toomer[6] himself says," there is no explicit evidence about the nature of the Hipparchus Table". Thus, there is no real proof that such a table ever existed.

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The only work of Hipparchus that has survived, namely his catalogue and commentary on stars, gives no account of his mathematical methods. Ptolemy ascribes to Hipparchus two numbers relating to the Moon's orbit, namely the ratios R/r and R/e, where R, r and e refer to the radius of the deferent, radius of the epicycle and the eccentricity of the lunar orbit respectively. Specifically, based on these two numbers, Toomer assumes the existence of a Table of Chords and proceeds to reconstruct this hypothetical table.

TOOMER'S RECONSTRUCTION

The chord of an angle is twice the sine of half of that angle, on a circle of unit radius, i.e., $\mathrm{crd}(\theta)=2\sin\left(\frac{\theta}{2}\right)$. Toomer uses the methods taken directly from the work of Āryabhaṭa. Āryabhaṭa uses a circle of radius 3438 units and uses angles at intervals of 3° 45' in his table of Rsines. Toomer creates a Chord Table in which the Chord lengths are 3438 times the one defined in the above equation, and tabulates them at intervals of 7°30′. If Toomer had taken directly the values from Āryabhaṭa's table of Rsines ($\theta/2=3°$ 45'), he would have reconstructed a Table of Chords entirely based on Āryabhaṭa. However, Toomer wants to show that there might have existed an independent Table of Chords, so he takes the values of sines from a modern table. To justify his reconstruction, Toomer proceeds to compute the two ratios ascribed to Hipparchus by Ptolemy. Since there is no knowledge of the mathematical methods of Hipparchus, Toomer assumes that Hipparchus might have used the same method as Ptolemy. On the basis of his reconstructed Table of Chords, as described above, and using the method of Ptolemy, Toomer computes the two numbers relating to the Moon's orbit, but gets them wrong. The value for the ratio R/r quoted by Ptolemy and ascribed to Hipparchus, is $\frac{3122\frac{1}{2}}{247\frac{1}{2}}$. The value obtained by Toomer for this ratio is $\frac{2913}{246\frac{1}{3}}$ which is quite far from the value of Hipparchus. Toomer then argues that Hipparchus must have made a mistake, tries a correction, but gets a value $\frac{3082\frac{2}{3}}{246\frac{1}{3}}$, which does not agree with that of Hipparchus either. But now, Toomer feels that the value is close enough to that of Hipparchus. He concludes therefore, that Hipparchus used a Chord Table of the proposed type and that, in addition, Hipparchus had committed the mistake as proposed by Toomer. Such is the nature of the "conclusive proof" provided by Toomer to show that Āryabhaṭa borrowed from Hipparchus. Toomer fares no better in calculating the value of the other ratio, R/e.

DISCUSSION

Toomer has constructed the Table of Chords based on a circle of radius of 3438 units of Āryabhaṭa and computes the two numbers relating to the Moon's orbit using the method of Ptolemy. Since the numbers so obtained do not agree with those given by Ptolemy (and ascribed to Hipparchus), the natural conclusion should have been that there is no relationship between the numbers of Hipparchus and those derived from Āryabhaṭa. However, in his zeal to prove the non-originality and the indebtedness of Āryabhaṭa to Hipparchus, Toomer further hypothesizes a particular mistake to have been committed by Hipparchus. Even after all this, still there is no agreement between the two sets of numbers. Yet, Toomer offers this as the conclusive proof that Āryabhaṭa borrowed from Hipparchus and the other scholars simply acclaim what Toomer says! It may be pointed out that the value of 3438 units for the radius of the base circle common in Indian astronomical texts has been grudgingly acknowledged[9] to be "presumably a development within Indian astronomy independent of Greek influence." Thus, there is no valid basis for the assertion that Āryabhaṭa derived his Table of RSines from a Table of Chords of Hipparchus.

On the other hand, Hayashi[10] has clearly demonstrated recently the originality of Āryabhaṭa's Rsine table. Hayashi has reexamined, on the basis of grammatically and mathematically precise interpretation of Nīlakaṇṭha, another verse in the second chapter of *Āryabhaṭīya*, verse 2.12, which gives second order differences in Rsines. While doing so, Hayashi has observed that six of the entries in the table of Rsines of Āryabhaṭa in verse 1.12 quoted earlier[7] differ from the correct values by one unit. These differences arise because of rounding off (*'ardhādhikena'*, rounded to the next integer because of its being greater than half) and could not have occurred if Āryabhaṭa had simply copied a table.

It may further be noted that while the use of R=3438 units for the radius of the base circle is common in Indian astronomy texts, other values are also used. For example, Varāhamihira uses R=120 units and lists values of Rsines at intervals of 3°45′ in his *Pañcasiddhāntikā*. One might suggest this to have been derived from a Greek chord table with R=60. While it is tempting to do so, the mere use of a sexagesimal base number alone, however, should not be taken as proof of borrowing from the Greek (cf. the earliest Vedic base was 360 days in a year).

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   makhi bhakhi phakhi dhakhi ṇakhi ñakhi *ṅakhi hasjha skaki kiṣga śghaki kighva*

ghlaki kigra hakya dhaki kica

sga śjha ṅva kla pta pha cha kalārdhajyāh //1.12//

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