INTRODUCTION

The *Tantrasaṃgraha* is a full-fledged text on Hindu Astronomy by Sri Nīlakaṇṭha Somayāji (AD 1444-1545) one of the eminent Kerala astronomers. In eight chapters it deals with all the important aspects of Hindu Astronomy and gives many mathematical, especially Trigonometric and Sphereical Trigonometric formulae, thereby vouchsafing for the deep knowledge of the author in this discipline. These are listed separately chapterwise in the course of this introduction for easy reference.

The text of the *Tantrasaṃgraha* is edited critically along with two commentaries *Yuktidīpikā* (for Chapters I-IV only) and *Laghuvivṛti* for the rest of the work by Dr. K.V. Sarma and published by Vishveshvaranand Vishva Bandhu Institute, Hoshiarpur in 1977. The English translation is based on the Sanskrit text of *Tantrasaṃgraha* of this edition by K.V. Sarma.

Nīlakaṇṭha, Author of Tantrasaṃgraha.

A detailed colophon occuring at the end of *Bhāṣya* by Nīlakaṇṭha himself on the Ganitapāda of the *Āryabhaṭīya* contains a good deal of information about him. According to the colophon it is determined that Nīlakaṇṭha belonged to the Gārgya gotra, was a follower of *Aśvalāyana-sūtra* of the *Ṛgveda* and was a Bhāṭṭa. He was the son of Jātavedas and had a younger brother named Śaṅkara.

From a Malayalam work entitled *Laghurāmāyaṇam* some more biographical details are obtained. He is said to be a resident of Kuṇḍagrāma, now known as Tṛk-kaṇṭi-yur near Tirur, Southern Railway, Ponnani Taluk South Malabar. The name of his Illam was Keḷallūr (Sanskritised into Keṛalā-sad-grāma). His wife was named Āryā and he had two sons Rāma and Dakṣiṇamurti. The great Malayalam poet Tuñcattu Ezhuṭṭacchan is said to have been a student of Nīlakaṇṭha.

From several references in his writings, it is known that Kaūṣītaki Āḍhya Netranārāyaṇa, known locally as Āzhvanceri Tampākkal patronised him and had great esteem for his erudition in Astronomy.

Nīlakaṇṭha studied Vedānta and some aspects of Astronomy under one Ravi. But the one who actually initiated him into the science of Astronomy and instructed him on the various principles underlying mathematical calculations was Dāmodara, son of the *Keṛala-Dṛgganita* author Parameśwara. Nīlakaṇṭha followed in the foot steps of Parmeśwara. and considered him as his Parama-guru.

Nīlakaṇṭha's writings.

The following works of Nīlakaṇṭha reflect his deep study of and ripe scholarship in astronomy.

1. *Golasāra*, embodies the basic astronomical elements and procedures.

2. *Siddhāntadarpaṇa* - a short work in thirty two anustubhs, enunciating the astronomical constants with reference to the Kalpa and specifying his views on the astronomical concepts and topics.

3. *Candracchāyāgaṇita*, - a short work in thirty two verses on the methods for the calculation of time from the measurement of the shadow of the gnomon cast by the Moon and vice-versa.

4. *Tantrasaṃgraha* in 432 verses and divided into eight chapters, is a major work of Nīlakaṇṭha and is an erudite treatise on Astronomy.

5. *Āryabhaṭīya - Bhāṣya* is an elaborate commentary on the cryptic and sūtra-like text of Āryabhaṭa, which comprehends in 121 āryās the fields of Mathematics and Astronomy.

6. *Siddhāntadarpaṇa - Vyākhyā* is a commentary on his own Siddhāntadarpaṇa.

7. A commentary on the *Candracchāyāgaṇita.*

8. *Grahanirṇaya,* a work on the computation of lunar and solar eclipses.

9. *Sundararāja-prasnottara.* Sundararāja son of Anantanārāyaṇa was an Astronomer of the Tamil Nadu and author of a detailed commentary on *Vākyakaraṇa.* He addressed Nīlakaṇṭha for clarification of certain points in Astronomy. Nīlakaṇṭha's detailed answers to these questions formed the above regular work.

10. *Grahaṇādi-grantha,* describes the necessity of correcting old astronomical constants by observations.

11. *Grahaparīkṣākrama* is a long tract of about 200 verses enunciating the principles and methods for verifying the astronomical computation by regular observations. Since these verses are found in *Nīlakaṇṭha's Bhaṣya* on the *Golapāda* of the *Āryabhaṭīya* it is not definitely known whether this is an independent work with the above title.

Date of Nīlakaṇṭha

Śankara, Nīlakaṇṭha's pupil, in his commentary on his teacher's *Tantrasaṃgraha* points out that the first and last verses of that work contain chronograms specifying the dates of the commencement and of the completion of the work. हे विष्णो निहितं कृत्स्नं (1680548) and लक्ष्मीशनिहितध्यान (1680553) are the Kali dates of the commencement and completion of the work. These dates work out to Kali year 4601, Mīna 26 and 4602, Meṣa both dates occuring in 1500.

In *Siddhānta-darpaṇa* and Nīlakaṇṭha's own commentary thereon give respectively, the year and actual date of his birth. He himself says he was born on the Kali day 16, 60, 181 which works out to 14th June 1944 AD. A contemporary reference made of him in a Malayalam work on astrology gives an evidence to the fact that Nīlakaṇṭha lived to a ripe old age, even to become a centenarian.

Versatality of Nīlakaṇṭha

Nīlakaṇṭha's writings substantiate his knowledge of the several branches of Indian philosophy and culture. Sundararāja, the Tamil astronomer, calls him *ṣaḍ-darśana-pāraṅgata,* 'one who had mastered the six systems of philosophy'.

In his writings he refers to a *Mīmāṃsa* authority, applies a grammatical dictum to establish a mathematical point, quotes extensively from Piṅgala's chandas-sūtra, scriptures, *Dharmaśāstras, Bhāgavata* and *Viṣhṇu Purāṇas* also.

As for Jyotiṣa works, he quotes from almost all important texts of all India prevalence, and uses all types of *Jyotiṣa* texts, *Gaṇita, Saṃhitā* and *Horā.* These texts are, *Vedāṅga-Jyotiṣa, Āryabhaṭīya,* Varahamihirā's *Pañcasiddhāntikā, Bṛhajjātaka* and *Bṛhatsaṃhitā, Sūryasiddhānta,* Sripati's *Siddhāntaśekara* and Munjala's *Laghumānasa, Parahitagaṇita* or *Grahacāranibandhana* of Haridatta, *Bhāṣya* by Bhāskara I on the *Āryabhaṭīya* and his *Laghu* and *Mahābhāskarīyas,* Govindasvāmī's *Bhāṣya* on the latter and Parameśvara's super-commentary thereon. Besides passages from his own teacher, Damodara, he quotes another Kerala author and a reputed astronomer of his times Mādhava often styled as Golavid.

MANUSCRIPT MATERIAL

Manuscripts of the Text of Tantrasaṃgraha (TS)

Twelve manuscripts, in all, have been used towards the critical edition of the textual verses.

A. Ms. 3810 of the Vishveshvaranand Institute Library, Hoshiarpur. This is a palmleaf manuscript, inscribed in Malayalam script, in 195 folios, 21 cm. × 3.5 cm., having 7 to 8 lines a page, with about 25 letters a line. It is written in two or three hands, the lettering of all of which is clear and shapely. The writing has been undergone the scrutiny of a reviser whose occasional corrections can be detected by their not being inked. The manuscript is in good preservation, though the corners have rounded off by frequent use. The manuscript is not dated nor any scribe mentioned, but its original repository is given as 'Vāraṇāsi', a reputed family of Nampūtiri brāhmins in Central Kerala.[1] At the close of the work, some miscellaneous matter has been inscribed on three folios. The manuscript contains the text of *Tantrasaṃgraha* and its commentary *Laghuvivṛti,* both complete and to a high degree of accuracy.

B. A paper transcript of a palmleaf manuscript in Malayalam script preserved in the Sanskrit College Library, Tripunithura (Kerala), Ms. No. 543-B, prepared by the late Rama Varma Maru Thampuran of the Cochin royal family in 1941 and later passed on by him to the present editor. The manuscript is not dated; neither has any scribe been mentioned. It contains a highly accurate text of the *Tantrasaṃgraha* with the commentary *Laghuvivṛti.* The codex contains also several short works on astronomy.

C. 1-10. Ten palmleaf manuscripts, all in Malayalam script, containing both the text and the commentary *Laghuvivṛti,* had been used in the preparation of the edition thereof through the *Trivandrum Sanskrit Series,*No. 188, (Trivandrum, 1958), and designated क to ञ. While the present edition of the textual verses is primarily based on the two highly reliable manuscripts A and B noticed above, the variant readings that occurred in the ten manuscripts and recorded in the said edition have been noted here with the *sigla* C_1 to C_{10}.

SUMMARY OF CONTENTS OF TANTRASAṂGRAHA

Chapter I Madhyama Prakaraṇam

Śloka

1. Invocation by the Author.

2-4. Civil day and sidereal day measures.

5, 6. Lunar and solar Months.

7-13 Intercalary month

14. Day of God.

15-18a Aeonic revolutions of the planets.

18b-22 Civil days in a *Yuga*

23-28a Calculaton of elasped Kalidays

26b-28a To find the mean position of planets from Ahargaṇa.

28b-29a Desantara Saṃskāra

29b Circumference of earth at latitude zero is given to be 3300 Yojanas.

30-34 Longitudinal time.

35-38a The Zero-positions of planets at the beginning of Kali

38b-40 Zero positions of the planets at the ninth minor *yuga*.

Chapter II Sphutaprakaraṇam

Śloka

1-3a Anomaly and order of the quadrants If $\alpha = 225'$ and $x = 925'$, the author gives R Sin x =

$$R \text{ Sin } 4\alpha + \frac{25(R \text{ Sin}5\alpha - R \text{ Sin}4\alpha)}{225}$$

In the reverse process given R Sin $\theta = x$, the formula for finding θ is given.

3b-13 These *ślokas* give the method to find the R sine of any arc between two R sines (R Sin kα and R Sin (K+1)α with better accuracy.

14-15a Give the method to compute the arc given its R Sin according to Mādhava. The use of the formula tan $\theta = \theta$ when θ is small is employed.

16 Rule of R Sin (A $\pm$ B) known as *Jivē paraspara Nyāya*

17-21 Given R Sin θ, to find θ. The formula Sin x =

$x - \frac{x^3}{6}$ when x is small is used.

21b-23a To determine position of the Sun using mandaphala and Śīghraphala.

24 The formula for declination of the sun is given

$$R \text{ Sin } \delta = \frac{R \text{ Sin } \lambda \text{ R. Sin } 24^\circ}{R}$$. Obviously

Nīlakaṇṭha has taken obliquity w to be equal to 24°

25-26b *Iṣṭa Koṭi* = $\frac{R\ \text{Cos}\ \omega \cdot R\ \text{Sin}\ \lambda}{R}$

Further he give R Sin α = $\frac{R\ \text{Cos}\ \omega\ R\ \text{Sin}\ \lambda}{R}\ \frac{R}{R\ \text{Cos}\ \delta}$

27b-28b Formula for Kṣitijyā is given as $\frac{12\ \text{Tan}\ \phi\ .\ \text{Sin}\ \delta}{12}$

The rest of the *ślokās* deals mainly with application of *cara saṃskāra* to the true position of planets and the measure of day and night after applying *cara saṃskāra.*

37 The rule *śiṣṭa cāpa gaṇa* is given to convert into arcs the *cara* and *jyā.*

40-43 These verses give the method to find the karṇa related to *Mandocca* and *Śīghrocca,* with and without successive approximation.

45-50 From the *mandakarṇa* the true sun is found out and the mean position from the true sun is to be obtined.

53-54 This gives the method to find instantaneous velocity of Sun and Moon.

Bibūtibhuṣan Datta and Awadesh Narayan Singh write, "Nīlakaṇṭha has made use of a result involving the differential of an inverse sine function. This result expressed in modern notation is

$$\{\text{Sin}^{-1}(e\ \text{Sin}\ \omega)\} = \frac{e\ \text{Cos}\omega\ d\omega}{\sqrt{1-e^2\sin^2\omega}}$$

(Reference : *Indian Journal of History of Science* **19**, (2) April 1984, p 100)

55-59 *Nakṣatra, Tithi, Karaṇa* and *Yoga* at the desired moment are given.

Chapter III Chāyā Prakaraṇam

1-5 After fixing the Gnonom, the drawing of East-West line and the North-South line are explained The correction to be made for drawing East-West line is given as $\frac{(R\ \text{Sin}\ \delta_1 - R\ \text{Sin}\ \delta_2)}{R\ \cos\ \phi}$ × *Karaṇa.*

Matsya-Karaṇa is employed for drawing North-South line.

9b 10a Formulae to find R Sin ϕ · R cos ϕ are given.

14-15 *Praṇas* of the rising of each sign at *Laṅka* and at any place.

16-21 *Iṣṭa śanku, Chāyā* and also *mahā śanku, mahāchāya* are given.

22-25 *Prāṇas* that have elapsed or yet to elapse.

26-28a Formulae corresponding to $Z = \delta \pm \phi$ are given.

28b Longitude of the sun is given as $R \text{ Sin } \lambda = \dfrac{R \text{ Sin } \delta . R}{R \text{ Sin } \omega}$

31-33 *Ayana Calana* is discussed. It is given to be equal to 27' for five divine years and works out to 54" for each civil year.

36-37 If E is the East-point and S, position of sun at

rising then $R \text{ Sin ES} = \dfrac{R \text{ Sin } \lambda . R \text{ Sin } \omega}{R \text{ Cos } \phi}$

30-40 The method to fix the directions from the shadow at a desired place/time is given.

41-46 The locus of the extremity of the shadow is given to be a Circle. It is the circum-circle of a particular triangle and the method this triangle is described.

The commentator of *Yukti Dīpika* clearly states that the locus being a circle is not established and is indicated only because of following the earlier teachers.

51 When $R \text{ Sin } \delta < R \text{ Sin } \phi$, then the *samamaṇdalà śanku* (Sun being on the prime vertical)

$$R \text{ Cos Z S} = \frac{R \text{ Sin } \delta \cdot R}{R \text{ Sin } \phi}$$

52 Sayaṇa longitude of the Sun from *sama maṇdala śaṅku* is given as

$$R \text{ Sin} = \frac{R \text{ Cos zs} - R \text{ Sin } \phi}{R \text{ Sin } \omega}$$

55b-57 To find the prāṇas elapsed or yet to elapse from *samamanḍala śaṅku.* The formula given is equivalent to, $h = \text{Sin}^{-1} \sqrt{R^2-x^2}$ where

$$x = \frac{\text{Sin } \delta}{\text{Sin } \phi} \cdot \text{Cos } \phi \cdot \frac{1}{\text{Cos } \delta}$$

59. *Kṣitijyā* $R \text{ Sin SK} = \dfrac{R \text{ Sin } \delta . R \text{ Sin } \phi}{R \text{ Cos } \phi}$

Daśa Praśṇas

The most important part of the work in so far as spherical trigonometrical results are concerned begin from śloka 60 and go right upto śloka 87. This portion deals with ten problems that arise when out of five astronomical constants three are given and the other two are to be found out Thus we have $5C_2 = 10$ such problems. The versatality of the author could be easily understood by studying this portion alone quite carefully. The five constants given are *śaṅku* (R Cosz), *nata* (R Sin h) *krānti* (R Sin δ) *dikagrā* (R Sin a) and *akṣa* (R Sin ϕ)

62-67 First Problem : Given Sin δ, Sin a and Sin ϕ. The author gives the following result (rendered in modern terminology).

$$\text{Cos z} = \frac{\text{Sin } \delta\text{-Sin } \phi \pm \sqrt{k^2\text{-sin}^2 \delta} \ . \ \sqrt{k^2\text{-Sin}^2 \phi}}{k^2}$$

where $k^2 = (\text{Sin a . Cos } \phi)^2 + \text{Sin}^2 \phi$

$$\text{Sin h} = \frac{\text{Sin z . Cos a}}{\text{Cos } \delta}$$

68-73 Second Problem : Given Sin h, Sin a, Sin ϕ The author introduces a term called *svadeśanata* which is R Sin ZM, is drawn perpendicular to PS from Z. The value for R Cos Z is obtained in a long drawn process. R Cos δ is obtained earily.

74-75a Third Problem : Given Sin h, Sin δ, Sin ϕ – Results 7 or the other two Cos z, Sin a are given easily, using the formula for Kṣitijyā.

75b-78a Fourth Problem : Given Sin h, Sin δ, Sin a, Sin ϕ is obtained in a long-drawn process.

Sin z is got directly using the result Sin h Cos δ = Sin z . Cos a

78b-79 Fifth Problem : Given Cos z, Sin a, Sin ϕ. Sin δ is obtained using cosine formula for Δ pzs.

Sin h is then easily got.

80-81a Sixth Problem : Given Cos z, Sin δ, Sin ϕ

Sin z, Sin a = (Sin δ ± Cos z Sin ϕ) ÷ Cos ϕ is established to obtain Sin a.

81b-83a Seventh Problem : Given Cos z, Sin δ, Sin a, Sin ϕ is obtained in a complicated manner by drawing a perpendicular SM from S to PZ.

83b-85 Eighth Problem : Given Cos z, Sin h, Sin ϕ. Using *svadeśanata* (R Sin ZM, ZM being perpendicular to PS) Cos δ is obtained in a long drawn process.

86 Ninth Problem : Given Cos z, Sin h, Sin a. R Sin δ is obtained easily.

87 Tenth Problem : Given Cos z, Sin h, Sin δ R Sin z . R Cos a = R Cos δ. R Sin h – a result that is used very often gives directly R Cos a.

88a Equinoctial shadow $s = \frac{12 \text{ R Sin } \phi}{\text{R Cos } \phi}$ is given,

91 Gives a formula for R Sin z

92 From R Sin z, by putting *agra*, a as 45', *Koṇa-Śaṅku* is obtained.

100b-104a *Prāglagna* and *Kālalagna* are given.

105-110 *Dṛkkṣepa* (Nonagesimal) lagna R Sin zv is calculated.

111-116 *Madhya Lagna* with and without the process of iteration is explained.

Chapter IV Lunar Eclipse

1-3 Moment of conjunction in Lunar Eclipse.

4-8a True Sun and Moon at sysygey.

8b-9a Radius of the orbit of the Moon is 34380 *yojanas*.

9b-10a Value π is taken to be equal to $\frac{354}{113}$ diameter of sun's disc is 4410 *yojanas* and that of the moon's disc is 315 *yojanas*.

10b-14a The true hypotenuse in *yojanās* of Sun and Moon is given.

15b-17a Diameter of the shadow of earth is given.

17b-19a Latitude and daily motion of the moon are given.

19b-42 These *ślokas* give in detail the possibility of the occurence of the eclipse, half duration of the eclipse, first and last contact and visibility of the eclipse.

The result corresponding to $\text{Cos } \theta = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24}$ is also employed.

43-46a *Akṣa valanam* and *Āyana valanam* are explained.

46b-53 The graphical representation of the eclipse is explained.

Chapter V Solar Eclipse

1-3a Parallax in longitude (*lambana*) and in latitude (*nati*) are given.

4-7 *Dṛggati* and *Dṛkkṣepjyā* are given from *Madhyajyā* and *Udayajyā*.

Dṛkkṣepajyā = R Sin zv = $\sqrt{\frac{(R \text{ Sin zm})^2 - (R \text{ Sin } \theta)^2}{R^2 - (R \text{ Sin } \theta)^2}} \cdot R$

(Refer fig‹ re under śloka 7.)

8-9 Parallax in longitude, *lambana*.

10-14 Parallax in Latitude, *nati*

15 Probability of the occurence of the eclipse, magnitude of total eclipse.

16.22a These verses give the middle of the eclipse and the correct way of applying the values of *lambana* to find the half duration.

22b-33 In these verses the method to determine the times of first and last contact by iteration are explained.

34-38 To determine the value of the disc at one's own place. Here *Chāyā* and *śaṅku* are given with reference to *Vitibha* or Nonagesimal V

R Sin zs = $\sqrt{\left(\frac{R \text{ Sin sv. } R \text{ Cos zv}}{R}\right)^2 + (R \text{ Sin sv})^2}$

Formula for *Dṛkkarṇa* is given in terms of R Sin ZS and R Cos ZS

39-49 These verses give the *Dṛkkarṇa* of the the Moon.

50-53a The middle of the eclipse.

53b-57a These verses deal with the non-visibility of the eclipse.

57b-63 Graphical representation of the eclipse.

Chapter VI Vyatīpāta prakaraṇam

1-2a Occurence of *Vyatīpāta, Vaidhṛti* and *Lāṭa*.

2b-6a R Sine declinations of Sun and Mooṇ Author

gives R Sin β = $\frac{R \text{ Sin } \lambda}{R} \cdot i$

Also for the Moon – true *krānti* is

$$\frac{R \text{ Sin } \beta \,.\, R \text{ Cos } \omega + R \text{ Cos } \beta \,.\, R \text{ Sin } \delta'}{R}$$

6b-12a An alternate method for the true *krānti* of the moon is given.

12b-13a Occurrence or otherwise of *Vyatīpāta*.

13b-24 These verses deal with the middle of *Vyatīpāta*, beginning and end of it and declare the inauspiciousness of the three types of *Vyatīpāta*.

Chapter VII Dṛkkarmaprakaraṇam.

1-4a Two types of *Dṛkkarṃa, Akṣa* and *Āyana Dṛkkarṃa* are given.

4b-9 Two methods for reduction to observation of the true planets are explained.

10-15 Visibility or otherwise of the rising and setting of the planets from the *Kālalagna*.

Chapter VIII Sṛṅgonnatiprakaraṇam

1-5 The true value of the motion of the Moon.

6-8 Celestial latitude and parallax in latitude.

9-17a To obtain the difference in the discs of the Moon and Sun.

17b-21 Orbal difference for computing the illuminated part of the moon.

22-29a Deflection (*valana*) of the illuminated portion.

29b-35a Graphical representation of the cusps.

36b-37a Time of Moon-rise after Sunset.

36-37a Orbits of the planets.

37b-40 Verification of the measure of the disc and conclusion of the *Grantha*, the Kali date of completion being given as लक्ष्मी शनिहित ध्यानै:
i.e. 16, 80, 553

नीलकण्ठसोमयाजिविरचितः

तन्त्रसंग्रहः

अथ प्रथमोऽध्यायः

[मङ्गलाचरणम्]

हे विष्णो निहितं कृत्स्नं जगत् त्वय्येव कारणे।
ज्योतिषां ज्योतिषे तस्मै नमो नारायणाय ते।। १ ॥

[सावननक्षत्रदिनमानम्]

रवेः प्रत्यग्भ्रमं प्राहुः सावनाख्यं दिनं नृणाम्।
आर्क्षमृक्षभ्रमं तद्वत्, ज्योतिषां प्रेरको मरुत्।। २ ॥

भ्रमणां पूर्यते तस्य नाडीषष्ट्या मुहुर्मुहुः।
विनाडिकापि षष्ट्यंशो नाड्या गुर्वक्षरं ततः[1]।। ३ ॥

प्राणो गुर्वक्षराणां स्याद् दशकं चक्रपर्यये।
खखषड्घनतुल्यास्ते वायुः समजवो यतः।। ४ ॥

[चान्द्रमासः]

पूर्वपक्षः शशाङ्कस्य विप्रकर्षो रवेः स्मृतः।
सन्निकर्षोऽपरः पक्षः सितवृद्धिक्षयौ ययोः।। ५ ॥

मासस्ताभ्यां[2] मतश्चान्द्रस्त्रिंशत्तिथ्यात्मकः स च।

[सौरमासः]

सौरोऽब्दो[3] भास्करस्यैव ज्योतिश्चक्रपरिभ्रम।। ६ ॥

मासस्तु राशिभोगः[4] स्यादयने चापि तद्गती[5]।

मूलम् - 1. B. omits the line.
2. B. अतः for मतः
3. B. सौराब्दो
4. A. राशिभागः
5. A. तद्गतिः

[अधिमासः]

त्रयोदशस्य चैत्रादिद्वादशानामियं भिदा।। ७॥

मेषाद्येकैकराशिस्फुटगतिदिनकृत्सङ्क्रमैकैकगर्भा-
श्चान्द्राश्चैत्रादिमासा इह न यदुदरे सङ्क्रमः सोऽधिमासः।
संसर्पः स्यात्, स चांहस्पतिरुपरि यदि ग्रस्तसङ्क्रान्तियुग्म-
स्तौ चाब्दर्त्वङ्गभूतौ सह सुचिरभवौ सोऽधिमासोऽत्र पश्चात्।। ८॥

अर्केन्द्वोः स्फुटतः[1] सिद्धास्त्रयो मासा मलिम्लुचाः[2]।
इति च ब्रह्मसिद्धान्ते मलमासास्त्रयः स्मृताः।। ९॥

द्वाभ्यां द्वाभ्यां वसन्तादिर्मध्वादिभ्यामृतुः स्मृतः।
मध्यादिभिस्तपस्यान्तैर्वर्षं द्वादशभिः स्मृतम्।। १०॥

त्रयोदशभिरप्येकं वर्षं स्यादधिमासके।
स्वोत्तरेणाधिमासस्य सम्बन्धो मुनिभिः स्मृतः।। ११॥

भानुना लङ्घितो मासो ह्यनर्हः सर्वकर्मसु।
षष्टिभिर्दिवसैर्मासः कथितो बादरायणैः।। १२॥

इति केषुचिदब्देषु सन्ति मासास्त्रयोदश।
श्रूयते [3]चर्तुयागादिष्वयमेव[4] त्रयोदश।। १३॥

[दिव्यदिनादिः]

दिव्यं दिनं तु सौरोऽब्दः[5], पितृणां मास ऐन्दवः।
सर्वेषां वत्सरोऽह्नां स्यात् षष्ट्युत्तरशतत्रयम्।। १४॥

[ग्रहादीनां युगपर्ययाः]

दिव्याब्दानां सहस्राणि द्वादशैकं चतुर्युगम्।
सूर्यस्य पर्ययास्तस्मादयुतघ्नरदार्णवाः।। १५॥

खाश्विदेवेशुसप्ताद्रिशराश्चेन्दोः, कुजस्य तु।
वेदाङ्गाहिरसाङ्काश्विकरा, ज्ञस्य स्वपर्ययाः।। १६॥

मूलम् - 1. A. B. अर्केन्दुस्फुटतः
2. A. मलिम्लुचः
3. B. C 1, 2, 5, 6 यागेयं मास एवं (B. यागेय)
4. A. omits the line haplographically.
5. A. B. सौराब्दः

नागवेदनभःसप्तरामाङ्कस्वरभूमयः।
व्योमाष्टरूपेदाङ्गपावकाश्च बृहस्पतेः।। १७ ॥

अष्टाङ्गदस्रनेत्राश्विखाद्रयो भृगुपर्ययाः।
भास्कराङ्गरसेन्द्राश्च शनेः, शश्यच्चपातयोः।। १८ ॥

नेत्रार्काष्टाहिवेदाश्च खखरामरदाश्विनः।

[युगे सावनदिवासादिः]

खखाक्षात्यष्टिगोसप्तस्वरेषुशशिनो युगे।। १९ ॥

सावना दिवसाश्चार्क्षा मार्ताण्डभगणाधिकाः।
अधिमासाः खनेत्राग्निरामनन्देषुभूमयः।। २० ॥

अयुतघ्नाब्धिवस्वेकशरा मासा रवेः स्मृताः।
खव्योमेन्दुयमाष्टाभ्रतत्त्वतुल्यास्तिथिक्षयाः।। २१ ॥

खखषण्णवगोनन्दनेत्रशून्यरसेन्दवः।
तिथयः, चान्द्रमासाः स्युः सूर्येन्दुभगणान्तरम्।। २२ ॥

[कलिदिनानयनम्]

द्वादशघ्नान् कलेरब्दान् मासैश्चैत्रादिभिर्गतैः।
संयुक्तान् पृथगाहत्याप्यधिमासैस्ततो हृतैः।। २३ ॥

सौरमासैर्युगोक्तैस्तैरधिमासैर्युतान् गतैः।
मासांश्च त्रिंशता हत्वा तिथीर्युक्त्वा गतः पृथक्।। २४॥

तिथिक्षयैर्निहत्यातो युगोक्ततिथिभिर्हृतान्।
अवमाञ्छोधयेच्छेषः सावनो द्युगणः कलेः।। २५ ॥

सप्तभिः क्षपिते शेषाच्छुक्रादिः स्याद् दिनाधिपः।

[देशान्तरसंस्कारः]

लङ्कामेरुगरेखायामुज्जयिन्यादितस्ततः ।। २८b ॥

पूर्वापरदिशोः कार्यं कर्म देशान्तरोद्भवम्।। २९a ॥

[देशान्तरकालः]

खखदेवा भुवो वृत्तं, त्रिज्याप्तं लम्बकाहतम्।। २९b ॥

स्वदेशजं, ततः षष्ट्या हृतं चक्रांशकाहतम्।
खखदेवहृतं भागाद्यन्तरं त्वङ्गभागयोः।। ३० ॥

स्वदेशसमयाम्योदग्रेखायां देशयोर्ययोः।
तदन्तरालदेशोत्थयोजनैः सम्मिते स्वके।। ३१ ॥

भूवृत्ते नाडिकैका स्यात् कालो देशान्तरोद्भवः।
निमीलनान्तरं यद्वा स्वदेशसमरेखयोः।। ३२ ॥

देशान्तरभवः काल, इन्दोरुन्मीलनादपि।। ३३a ॥

[देशान्तरकालस्य धनर्णत्वम्]

प्रागेव दृश्यते प्रत्यक्, पश्चात् प्राच्यां ग्रहः सदा।। ३३b ॥

देशान्तरघटीक्षुण्णा मध्या भुक्तिर्द्युचारिणाम्।
षष्ट्या भक्तमृणं प्राच्यां रेखायाः, पश्चिमे धनम्।। ३४ ॥

[ग्रहाणां कल्यादिध्रुवाः]

–4° 45' 46''
षड्वेदेष्वब्धिवेदास्तु विलिप्तादिध्रुवो विधोः।
–3ʳ 29° 17' 5''
प्राणात्यष्ट्यङ्कनेत्राग्नितुल्यं चन्द्रोच्चमध्यमम्।। ३५ ॥

–11ʳ 17° 47'
सप्तसागरशैलेन्द्रभवा लिप्तादयोऽसृजः।
–36'
षट्त्रिंशल्लिप्तिकाः शोध्या विदो, जीवेतु योजयेत्।। ३६ ॥

+12° 10'
पङ्क्त्यर्कतुल्यलिप्तादि, सिते राशिः षडंशकाः।
+1ʳ 6° 13' +11ʳ 17° 20'
विश्वतुल्याः कलाश्च स्वं, नखात्यष्टिभवाः शनेः।। ३७ ॥

+6ʳ 22° 20'
पाते तु मण्डलाच्छुद्धे नखाकृतिरसा अपि।। ३८a ॥

[नवमयुगादौ[1] ध्रुवाः]

कल्यादिध्रुवका ह्येते युगभोगसमन्विताः ।। ३८b ।।

576

तत्तद्युगे ध्रुवा ज्ञेयाः, षडश्वेष्वब्दकंयुगम् ।

7500

भगणात् खखभूताश्वैर्युगभोगस्त्ववाप्यते ।
अष्टघ्नयुगभोगाः स्वमतः कल्यादिजे ध्रुवे ।। ३९ ।।

[ग्रहाणां मन्दोच्चाः]

127' 220' 172' 80' 240'

स्वरखयः खाकृतयो द्विनगभुवोऽशीतिरभ्रजिनाः ।

78'

भौमान्मन्दोच्चांशा, वसुतुरगा भास्करस्यापि ।। ४० ।।

[।। इति तन्त्रसंग्रहे मध्यमप्रकरणं नाम प्रथमोऽध्यायः ।।]

अथ द्वितीयोऽध्यायः

स्फुटप्रकरणम्

[केन्द्रं पदव्यवस्था च]

स्वोच्चोनो विहगः केन्द्रं, तत्र राशित्रयं पदम् ।
ओजे पदे गतैष्याभ्यां बाहुकोटी, समेऽन्यथा ।। १ ।।

[ज्याग्रहणं चापीकरणं च]

225

लिप्ताभ्यस्तत्त्वनेत्राप्ता गता ज्याः, शेषतः पुनः ।

1. This refers to a contemporary date of the author Nīlakaṇṭha Somayāji (born A.D. 1444), who takes 576 years as a 'minor' *yuga* (verse 39, below). Thus, eight *yugas* (8×576 = 4608 years) from the beginning of Kali will end and the ninth *yuga* will commence at the close of the Kali year 4608 (A.D. 1507-8).

225
गतगम्यान्तरघ्नाच्च हृतास्तत्त्वयमैः क्षिपेत्।। २ ।।

दोःकोटिज्ये नयेदेवं ज्याभ्यश्चापं विपर्ययात्।। ३a ।।

[चापसन्धिगतार्धज्याः]

विलिप्तासदशकोना ज्या राश्यष्टांशधनुःकलाः।। ३b ।।

223½
आद्यज्यार्धात् ततो भक्ते सार्धदेवाश्विभिस्ततः।
त्यक्ते द्वितीयखण्डज्या द्वितीया ज्या च तद्युतिः।। ४ ।।

ततस्तेनैव हारेण लब्धं शोध्यं द्वितीयतः।
खण्डात् तृतीयखण्डज्यां[1] द्वितीयस्तुद्युतो गुणः।। ५ ।।

तृतीयः स्यात् ततश्चैवं चतुर्थाद्याः क्रमाद् गुणाः।। ६a ।।

[अर्धज्यानयने प्रकारान्तरम्]

व्यासार्धं प्रथमं ततो वान्यान् गुणान् नयेत्।। ६b ।।

113 21600 355
त्रीशघ्नचक्रलिप्ताभ्यो व्यासोऽर्थेष्वग्निभिर्हृतः।
तद्दलाद्यज्ययोः कृत्योर्भेदान्मूलमुपान्तिमा।। ७ ।।

अन्त्योपान्त्यान्तरं द्विघ्नं गुणो व्यासदलं हरः।
आद्यज्याया[2] स्तथापि स्यात् खण्डज्यान्तरमादितः।। ८ ।।

ताभ्यां तु गुणाहाराभ्यां द्वितीयादेरपि क्रमात्।
उत्तरोत्तरखण्डज्याभेदाः पिण्डगुणार्धतः।। ९ ।।

एवं सावयवा जीवाः सम्यङ्नीत्वा पठेत् क्रमात्।। १०a ।।

[इष्टप्रदेशे सूक्ष्मज्याः]

इष्टदोःकोटिधनुषोः स्वसमीपसमीरिते।। १०b ।।

ज्ये द्वे सावयवे न्यस्य कुर्यादूनाधिकं धनुः।
13751
द्विघ्नतल्लिप्तिकाप्तैकशरशैलशिखीन्दवः।। ११ ।।

मूलम् - 1. A. खण्डः स्यात्; B. खण्डस्य 2. C. आद्यज्यातः

न्यस्यात् छेदाय च मिथस्तत्संस्कारविधित्सया।
छित्वैकां प्राक् क्षिपेज्जह्यात् तद्धनुष्यधिकोनके ।। १२ ।।

अन्यस्यामथ तां द्विघ्नां तथाऽस्यामिति संस्कृतिः।
इति ते कृतसंस्कारे[1] स्वगुणो धनुषोस्तयोः ।। १३ ।।

तत्राल्पीयः कृतिं त्यक्त्वा पदं त्रिज्याकृतेः परः[2] ।। १४a ।।

[इष्टज्यायाः माधवोक्तं चापीकरणम्]

ज्ययोरासन्नयोर्भेदभक्तस्तत्कोटियोगतः ।। १४b ।।

छेदस्तेन हृता द्विघ्ना त्रिज्या तद्धनुरन्तरम् ।। १५a ।।

[माधवोदितं ज्याचापानयनम्]

इति ज्याचापयोः कार्यं ग्रहणं माधवोदितम्।
विधान्तरं च तेनोक्तं तयोः सूक्ष्मत्वमिच्छताम् ।। १५ ।।

['जीवे परस्पर' -न्यायः]

जीवे परस्परनिजेतरमौर्विकाभ्या-
मभ्यस्य विस्तृतिदलेन विभाज्यमाने।
अन्योन्ययोगविरहानुगुणे भवेतां
यद्वा स्वलम्बकृतिभेदपदीकृते द्वे ।। १६ ।।

शिष्टचापघनषष्ठभागतो
विस्तरार्धकृतिभक्तवर्जितम् ।।
शिष्टचापमिह शिञ्जिनी भवेत्
स्पष्टता भवति चाल्पतावशात् ।। १७ ।।

[इष्टज्यानयनम्]

ऊनाधिकधनुर्ज्यां च नीत्वैवं पठितां न्यसेत् ।। १८ ।।

ऊनाधिकधनुः कोटिजीवया तां समीपजाम्।
निहत्य पठितां तस्याः कोट्या शिष्टगुणं च तम् ।। १९ ।।

तद्योगं वाथ विश्लेषं हरेद् व्यासदलेन तु।
इष्टज्या भवति, स्पष्टा तत्फलं स्यात् कलादिकम् ।। २० ।।

मूलम् - 1. A. B. संस्काराः 2. A. परम्

न्यायेनानेन कोट्याश्च मौर्व्याः कार्या सुसूक्ष्मता।। २१a ॥

[रविस्फुटः]

त्र्यभ्यस्तबाहुकोटिभ्यां अशीत्याप्ते फले उभे।। २१b ॥

चापितं दोःफलं कार्यं स्वर्णं सूर्यस्य मध्यमे।
केन्द्रोर्ध्वार्धे च पूर्वार्धे तत्कालार्कस्फुटः, स च।। २२ ॥

मध्यसावनसिद्धोऽतः कार्यः स्यादुदये पुनः।। २३a ॥

[चरप्राणाः]

संस्कृतायनभागादेर्दोर्ज्या कार्या रवेस्ततः।। २३b ॥

चतुर्विंशतिभागज्याहतायास्त्रिज्यया हृतः।
अपक्रमगुणोऽर्कस्य तात्कालिक इह स्फुटः।। २४ ॥

तत्त्रिज्याकृतिविश्लेषान्मूलं द्युज्याथ कोटिका।
दोर्ज्यापक्रमकृत्योश्च भेदान्मूलमथापि वा।। २५ ॥

अन्त्यद्युज्याहता दोर्ज्या त्रिज्याभक्तेष्टकोटिका।
त्रिज्याघ्नेष्टद्युजीवाप्ता चापितार्कभुजासवः।। २६ ॥

दोःप्राणलिप्तिकाभेदमविनष्टं तु पालयेत्।
विषुवद्भाहता क्रान्तिः सूर्याप्ता क्षितिमौर्विका।। २७ ॥

त्रिजयाघ्नेष्टद्युजीवाप्ता चापिता स्युश्चरासवः।। २८a ॥

[रवेर्गतिकलाः]

लिप्ताप्राणान्तरं भानोर्दोःफलं च चरासवः।। २८b ॥

स्वर्णसाम्येन संयोज्या भिन्नेन तु वियोजयेत्।
भानुमध्यमभुक्तिघ्नं चक्रलिप्ताहृतं फलम्।। २९ ॥

भानुमध्ये तु संस्कार्यं स्फुटभुक्त्याहतं स्फुटे।। ३०a ॥

[ग्रहेषु चरस्य संस्कारप्रकारः]

उदक्स्थेऽर्के चरप्राणाः शोध्याः स्वं याम्यगोलगे।। ३०b ॥

व्यस्तमस्ते तु संस्कार्या, न मध्याह्नार्धरात्रयोः।
युग्मौजपदयोः स्वर्णां रवौ प्राणकलान्तरम्।। ३१ ॥

दोःफलं पूर्ववत्कार्यं रवेरेभिर्द्युचारिणाम्।
मध्यभुक्तिं स्फुटां वापि हत्वा चक्रकलाहृतम्।। ३२ ।।

स्वर्णं कार्यं यथोक्तं, तद् व्यस्तं वक्रगतौ स्फुटे।। ३३a ।।

[चरसंस्कारेण दिनरात्रिमानम्]

अहोरात्रचतुर्भागे चरप्राणान् क्षिपेदुदक्।। ३३b ।।

याम्ये शोध्या दिनार्धं तद् रात्र्यर्धं व्यत्ययाद् भवेत्।
दिनक्षपे द्विनिघ्ने ते चन्द्रादेः स्वैश्चरासुभिः।। ३४a ।।

[चन्द्रस्फुटः]

इन्दूच्चयोः स्वदेशोत्थरव्यानीतचरादिजम्।
संस्कारं मध्यमे कृत्वा स्फुटीकार्यो निशाकरः।। ३५ ।।

दोःकोटिज्ये तु सप्तघ्ने अशीत्याप्ते फले उभे।
चापितं दोःफलं कार्यं स्वमध्ये स्फुटसिद्धये।। ३६ ।।

[चरज्यादीनां चापीकरणम्]

ज्याचापान्तरमानीय शिष्टचापघनादिना।
युक्त्वा, ज्यायां धनुः कार्यं पठितज्याभिरेव वा।। ३७ ।।

118, 18, 103
त्रिखरूपाष्टभूनागरुद्रैः त्रिज्याकृतिः समा।
एकादिघ्न्या दशाप्ताया घनमूलं, ततोऽपि यत्।। ३८ ।।

तन्मितज्यासु योज्याः स्युरेकद्व्याद्या विलिप्तिकाः।
चरदोःफलजीवादेरेवमल्प[1] धनुर्नयेत्।। ३९ ।।

[मन्दशीघ्रकर्णौ]

आद्ये पदे चतुर्थे च व्यासार्धे कोटिजं फलम्।
युक्त्वा त्यक्त्वान्ययोस्तद्दोःफलवर्गैक्यजं पदम्।। ४० ।।

कर्णः स्यादविशेषोऽस्य कार्यो मन्दे[2], चले न तु।

मूलम् - 1. A. मल्पं 2. B. मान्दे

[मन्दकर्णः]

दोःकोटिफलनिघ्नाद्यो[1] कर्णात् त्रिज्याहृते फले ।। ४१ ।।

ताभ्यां कर्णाः पुनः साध्यो भूयः पूर्वफलाहतात्।
तत्तत्कर्णात् त्रिभज्याप्तफलाभ्यामविशेषयेत् ।। ४२ ।।

[मन्दकर्णे प्रकारान्तरम्]

विस्तृतिदलदोः फलकृति-
वियुतिपदं कोटिफलविहीनयुतम्।
केन्द्रे मृगकर्किगते
स खलु विपर्ययकृतो भवेत् कर्णः ।। ४३ ।।

तेन हृता तिज्याकृति-
रयत्नविहितोऽविशेषकर्णः स्यात्।
इति वा कर्णः साध्यो
मान्दे सकृदेव माधवप्रोक्तः ।। ४४ ।।

[मन्दकर्णेन रविस्फुटः]

त्रिज्याघ्नो दोर्गुणः कर्णभक्तः[2] स्फुटभुजागुणः ।
तद्धनुः संस्कृतं स्वोच्चं नीच्चं वा युक्तितः स्फुटम् ।। ४५ ।।

[रविस्फुटाद् ग्रहमध्यमः]

अर्कस्फुटेनानयनं प्रकुर्यात्
स्वमध्यमस्यात्र वितुङ्गभानोः ।
भुजागुणं कोटिगुणं च कृत्वा
मृगादिकेन्द्रेऽन्त्यफलाख्यकोट्योः[3] ।। ४६ ।।

भेदः, कुलीरादिगते तु योग-
स्तद्वर्गयुक्ताद् भुजवर्गतो यत्।
पदं विपर्यासकृतः स कर्णः,
त्रिज्या कृतेस्तद्विहृतस्तु[4] कर्णः ।। ४७ ।।

मूलम् - 1. A. B. निघ्नाद् यत्
2. C. कर्णः भक्तः
3. A. B. $C_{3,9}$ कोट्या
4. C_6 विहृतः स कर्णः

तेनाहतामुच्चविहीनभानो-
जीवां भजेत् व्यासदलेन, लब्धम्।
स्वोच्चे क्षिपेच्चापितमाद्यपादे,
चक्रार्धतः शुद्धमपि द्वितीये।। ४८ ॥

चक्रार्धयुक्तं तु तृतीयपादे,
संशोधितं मण्डलतश्चतुर्थे।
एवं कृतं सूक्ष्मतरं हि मध्यं
पूर्वं पदं यावदिहाधिकं स्यात्।। ४९ ॥

अन्त्यात् फलात्[1] कोटिगुणं[2] चतुर्थं
त्वारभ्यते[3] यद्यधिकात्र कोटिः।
सर्वज्ञ विष्कम्भदलं श्रुतौ वा
व्यासार्धके स्याद् विपरीतकर्णः।। ५० ॥

[स्फुटान्मध्यमे प्रकारान्तरम्]

अर्केन्द्वोः[4] स्फुटतो मृदूच्चरहिताद् दोःकोटिजाते फले
नीत्वा, कर्किमृगादितो विनिमयेनानीय कर्णं सकृत्।
त्रिज्यादोःफलघाततः श्रुतिहृतं चापीकृतं, तत् स्फुटे
केन्द्रे मेषतुलादिगे धनमृणं तन्मध्यसंसिद्धये।। ५१ ॥

[मन्दकर्णे प्रकारान्तरम्]

मध्यतः स्फुटतश्चोच्चमुज्झित्वा तद्भुजे उभे।
गृहीत्वाद्यां तयोस्त्रिज्याहताऽन्याप्ता, श्रुतिः स्फुटा[5]।। ५२ ॥

[रविचन्द्रयोः तत्कालस्फुटः]

चन्द्रबाहुफलवर्गशोधितत्रिज्यकाकृतिपदेन संहरेत्।
तस्य कोटिफललिप्तिकाहतां केन्द्रभुक्तिमिह, यच्च लभ्यते।। ५३ ॥

तद्विशोधय मृगादिके[6] गतेः,[7] व्रिप्यताहिम तु कर्कटादिके[8]।
तद् भवेत् स्फुटतरा गतिर्विधोरस्य तत्समयजा, रवेरपि[9]।। ५४॥

मूलम् - 1. $C_{1.2.6.8}$ पदात्
2. B.$C_{.3.4.8}$ गुणः
3. $C_{3.8.}$ चारभ्यते
4. B. अर्केन्दोः; C. अर्केन्दु
5. A. स्फुटः
6. C_6 मृगादिगे
7. C_6 गतौ
8. A. C_{6-9} कर्कटादिये
9. C_{10} विधोरपि

[तत्कालनक्षत्रम्]

लिप्तीकृतो निशानाथः शतैर्भाज्योऽष्टभिः, फलम्।
अश्विन्यादीनि भानि स्युः, षष्ट्या हत्वा गतागते।। ५५ ॥

गतगन्तव्यनाड्यः स्युः स्फुटभुक्त्योदयावधेः।

[तत्कालतिथिः]

अर्कहीनो निशानाथो लिप्तीकृत्य विभज्यते।। ५६ ॥

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शून्याश्विपर्वतैर्लब्धास्तिथयो या गताः क्रमात्।

[तत्कालकरणम्]

भुक्त्यन्तरेण नाड्यः स्युः षष्ट्या हत्वा गतागते।। ५७ ॥

तिथ्यर्धहारलब्धानि करणानि बबादितः।
विरूपाणि सिते पक्षे सरूपाण्यसिते विदुः।। ५८ ॥

[तत्कालविष्कम्भः]

विष्कम्भाद्या रवीन्द्वैक्याद्योगाश्चाष्टशतीहृताः।
भुक्तियुक्त्या गतैष्याभ्यां षष्टिघ्नाभ्यां च नाडिकाः।। ५९ ॥

[कुजादिस्फुटः]

मान्दं शैघ्रं पुनर्मान्दं शैघ्रं चत्वार्यनुक्रमात्।
कुजगुर्वर्कजानां हि कर्माण्युक्तानि सूरिभिः।। ६० ॥

[स्फुटकर्म]

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दोःकोटिज्याष्टमांशौ स्वखाब्ध्यंशोनौ शनेः फले।
दोर्ज्या त्रिज्यासमैक्यं गुणो मान्दे कुजेड्ययोः।। ६१ ॥

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नवाग्नयो द्व्यशीतिश्च हारौ दोःकोटिजीवयोः।
पृथक्स्थे मध्यमें कार्यं दोःफलस्य धनुर्दलम्।। ६२ ॥

रविमध्यं विशोध्यास्मात् पृथक्स्थाद् बाहुकोटिके।
आनीय बाहुजीवायास्त्रिज्याप्तं गुरुमन्दयोः।। ६३ ॥

षोडशभ्यो नवभ्यश्च, कुजस्यापि स्वदोर्गुणात्।

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त्रिज्याप्तं द्विगुणां शोध्यं त्रीषुभ्यः शिष्यते गुणः।। ६४॥

अशीतिरेव तेषां हि हारस्ताभ्यां फले उभे।
आनीय, पूर्ववत् कर्ण सकृत् कृत्वा, थ[1] दोःफलम्।। ६५॥

त्रिज्याघ्नं कर्णभक्तं यत् तद् धनुर्दलमेव च।
मध्यमे कृतामान्दे तु संस्कृत्यातो विशोधयेत्।। ६६॥

मन्दोच्चं तत्फलं कृत्स्नं कुर्यात् केवलमध्यमे।
तस्मात् पृथक्कृता[2]च्छैघ्रं प्रागवदानीय चापितम्।। ६७॥

कृतामान्दे तु कर्तव्यं[3] सकलं, स्यात् स्फुटः स च।। ६८a॥

[बुधशुक्रयोः स्फुटः]

बुधमध्यात् स्वमन्दोच्चं त्यक्त्वा दोःकोटिजीवयोः।। ६८b॥

षडंशाभ्यां फलाभ्यां तु कर्णः कार्योऽविशेषितः[4]।
दोःफलं केवलं स्वर्णं केन्द्रे जूकक्रियादिगे।। ६९॥

एवंकृतं हि[5] यन्मध्यं स्फुटमध्यं बुधस्य तु।
रविमध्यं ततः शोध्यं, दोःकोटिज्ये ततो नयेत्।। ७०॥

दोर्ज्या द्विघ्ना त्रिभज्याप्ता शोध्यैकत्रिंशतो[6] गुणः।
मन्दकर्णहतः सोऽपि त्रिज्याप्तः स्यात् स्फुटो गुणः।। ७१॥

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तद्धते बाहुकोटिज्ये खाहिभक्ते फले उभे।
ताभ्यां कर्णं सकृन्नीत्वा त्रिज्याघ्नं दोःफलं हरेत्।। ७२॥

कर्णेनाप्तस्य यच्चापं कृत्स्नं तद् भानुमध्यमे।
क्रमेण प्रक्षिपेज्जह्यात् केन्द्रे मेषतुलादिगे।। ७३॥

एवं शीघ्रफलेनैव संस्कृतं रविमध्यमम्।
बुधः स्यात् स स्फुटः, शुक्रोऽप्येवमेव स्फुटो भवेत्।। ७४॥

मूलम् - 1. C_1 कृत्वा तु
2. C_{10} पृथक्स्थितात्
3. C1 संस्कार्य for कर्तव्यं
4. $C_{1-5.10}$ कार्योऽविशेषतः
5. A. $C_{6.7}$ तु for हि
6. C_1 त्रिंशको

[शुक्रे विशेषः]

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मन्दकेन्द्रभुजाजीवा खजिनांशेन संयुता।
14
मनवस्तस्य हारः स्यात्, तद्भक्ते बाहुकोटिके।। ७५ ॥

स्यातां, मन्दफले तस्य दोःफलं च स्वमध्यमे।
[1]कृत्वाऽविशेषकर्णं च,[2] क्रियतां शीघ्रकर्म च।। ७६ ॥

द्विघ्ना दोर्ज्या त्रिभज्याप्ता त्याज्या[3]ऽस्यैनषष्टितः।
गुणः, सोऽपि स्फुटीकार्यो मन्दकर्णेन पूर्ववत्।। ७७ ॥

गुणः स मन्दकर्णघ्नस्त्रिज्याप्तस्तस्य च स्फुटः।
अशीत्याप्ते भुजाकोटी, तद्घ्ने शीघ्रफले भृगोः।। ७८ ॥

दोःफलं त्रिज्यया हत्वा शीघ्रकर्णहृतं भृगोः।
चापितं भास्वतो मध्ये संस्कुर्यात् सः स्फुटः सितः।। ७९ ॥

[ग्रहाणां दिनभुक्त्यानयनम्]

श्वस्तनेऽद्यतनाच्छुद्धे वक्रभोगोऽवशिष्यते।
विपरीतविशेषोत्थश्चारभोगस्तयोः स्फुटः।। ८० ॥

मूलम् - 1. C_1 कृता for कृत्वा
2. A. C_{1-6} कर्णश्च
3. $C_{1-5,10}$ शोध्या for त्याज्या

Chapter I
Madhyamā Prakaraṇam

Invocation by the Author

1. Oh Viṣṇu! This entire world shines because of you only. Salutations to you, o Narayana who is the light of all things that shine.

Commentator Śankara Vāriar states that the first foot of this verse gives the *Kali* date of the beginning of the work. K.V. Sarma states this date to 16,80,548. The *Kali* date of completion of the work is given is the last verse of the work which works out to 16,80,553. These dates work out to *Kali* year 4601, *Mīna* 26 and 4602, *Meṣa*, both dates occuring in 1500 AD (*Tantrasaṃgraha*, Edited by K.V. Sarma, p xxxv).

Civil day and Sidereal day Measures :

2-4. The revolution of the Sun towards the west (from east) is termed a civil day, a day of human beings; in the same manner; the revolution of a star is a sidereal day; the *marut* (wind) is the impeller of all the shining objects. Its (of wind) revolution is completed (once in 60 *nāḍīs* again and again. A *vināḍī* is one-sixtieth part of *nādi* and one-sixtieth part of a *vinādi* is a *guruvakṣara;*Ten such *guruvakṣaras* is called a *prāṇa;* thus one revolution is equal to 216,00 (*kha kha ṣaḍghna*) *prāṇas,* since the wind has constant speed always.

Lunar Month

5, 6a. The (period of) separation of the moon from the sun, (from the time the Moon was in conjunction with the Sun) is termed the earlier forthight; (after the full-Moon time) and the approach towards (the sun) is called the later fortnight. Of these two fortnights (during which) the illuminated portion (of the Moon) gradually increases and decreases, a lunar month is known; and it consists of 30 *tiṭhis* (lunar days).

Solar Month

6b. 7a. A year is only the (period) revolution of the Sun in the zodiac. A solar month is the time elapsed in a zodical sign. The two *āyanas* are only the movement of the sun with regards to the northern and southern directions.

Intercalary month.

7b. The following is the difference between the twelve months (named from) *chaitra* and thirteenth month.

8. Lunar months named from *caitra* are those that enclose one *Saṅkramaṇa* which is the crossing by true sun of each of the *rasis* from *meṣa* etc.,

That is an intercalary month which does not enclose such a crossing of the sun.

Such a month (which does not enclose a crossing) will be called a *saṃsarpa* if it is followed by *aṃhaspati*,a month which encloses two crossings. These two which invariably occur together are taken as part of the twelve month year and seasons. These two which invariably occur together are taken as part of the twelve month year and seasons. (The lunar month without a crossing) that ocurs later is indeed the intercalary month.

9. The three months (*Saṃsarpa, Amhaspati* and *Adhimāsa*) that are determined from the true sun and moon, are called *malimulcuca,* or impure. Thus in *Brahma-Siddhānta* these three are mentioned as *mala māsas.*

10-11. By combining the months from *madhu* onwards two by two, the seasons like spring (*Vasanta*) are determined. From the month *madhu* to the end of *tapasyā*, a year is determined. A year can also have 13 months if there is an intercalary month. It has been declared by the *riṣis* that it (the name of *adhimāsa*) is related to the month that is following it.

12-13. The (lunar) month that is jumped over by the sun (i.e. in which there is no *saṅkramana*) is not fit for all auspicious activities. The *adhimāsa* together with the following lunar month was considered as a single month of 60 *tithis* by the followers of Bādarāyaṇa.

13. (In this way as we have explained) in some years there are thirteen months. The same is called *trayodaśa* or the thirteenth (in the Vedās) while talking of the seasons and sacrifices.

Day of god.

14. A day of gods is one solar year; (a day) of the *pitṛs* is a lunar month. For all one year is of 360 days (in their own measures).

Aeonic revolutions of the planets.

15. A *caturyuga* consists of 12,000 years of ten gods. Hence the revolutions of the sun will be 432 multipled by 10^4 (*ayuta*).

16. The revolutions of Moon is 5,77,53,320; of Mars is 22,96,864; of Mercury's

own revolutions is 1,79,37,048; of Jupiter is 3,64,180; the number of revolutions of Venus is 70,22,268; of Saturn is 1,46,612; the apogee of the Moon is 4,88,122 and of the nodes (Pata) is 2,32,300.

Civil days in a Yuga

18b-22. In yuga the number of Civil days is 157,79,17,200; the number of sidereal days is increased by the number of sidereal revolutions of the sun (43,20,000).

The number of intercalary months is 15,93,320; the number of solar months are stated as the product of 5148 by 10,000 (ayuta); number of elapsed *tithis* is equal to 2,50,82,100; number of tithis is 1,69,29,99,600;

The number of lunar months is the difference between the revolutions of the Sun and Moon.

Note : (i)

Moon	5,77,53,320
Sun	43,20,000
∴ Lunar Month :	53433320

(ii) Nīlakaṇṭha has specifically used the word *Svaparyāyā* their own revolutions, for *Budha* and *Śukra*. As the commentator Śankara Vāriar explains Nīlakaṇṭha has departed from the older model, where these revolutions were attributed to the so called *Śīgrocca* of *Budha* and *Śukra*. This point will be made clear later.

Computation of elapsed Kalidays or ahargaṇa

23-24 (a) Multiply the number of years (x) that have elapsed from the beginning of *Kaliyuga* by 12; To that add the lunar months (y) that have elapsed from *Caitra* in the currenty year. Keep the result separately (12x+y). Multiply this by the number of Intercalary months in a *yuga* (15,93,320). Divide the result by the number of solar months in a *yuga* (5,18,40,000).

$$\text{i.e. we get } z = \frac{(12\ x + y)\ 15,93,320}{5,18,40,000}$$

This is the number of *adhimāsas* elapsed since the beginning of *Kaliyuga*.

24b. To this (Z) add the months that is kept separate (12 x + y). These (lunar) months are multiplied by 30, and to the result add the number of *tithis* that have elapsed (in the current month). Keep the result separate. We then have

L = (12 x +y + z) 30 + t (number of *tithis* elapsed)

25-26a. Multiply the result by the number of *tithis* omitted in a *yuga*. The result is the *avama dinas* (K) (omitted lunar *tithis*).

$$K = \frac{L \times 250, 82, 100}{160, 29, 99, 600}$$

Subtract this (K) from the result kept separate (This gives L-K). The result is the number of civil days (*dyugaṇa*) that have elapsed from the epoch, *Kaliyuga*. Dividing this by seven, and by calculating from Friday, the remainder gives the lord of the day and thus the week day is got.

To find the mean position of planets from the ahargaṇa

26b. From the number of Kali days (obtained as set forth earlier), by multiplying it with the number of revolutions (of each planet) ann dividing the result by the revolutions of civil days in a *yuga*, (We get the number of revolutions) that are gone.

27.28.a There itself, the remainder is multiplied by 12 (and divided by Civil days in a *yuga*), and the number of rāṣi is obtained; Again (the subsequent remainder being multiplied by 30, 60 and divided by bhoodiṇa) give the number of degrees, minutes etc., the results added with the so called *dhṛuva* values (positions) of each planet at the beginning of *Kaliyuga* give the mean position of each planet on that day at the time of mean sunrise (at Lanka).

Correction for longitude (Deśāntara saṃskāra)

28b. 29.a. From the meridian line passing through Lanka, Mēru and Ujjain, the correction arising due to *Deśāntara* (due to the longitude of a place) is to be done for the places to the East and to the West.

The method to find longitudinal time

29b. The circumference of the earth (at a place whose latitude is Zero) is 3300 (*yojanas*); This is divided by R and multiplied by the R Cosine of the latitude (*lambaka*) of the place. The result is the rectified circumference

$$\text{at the place i.e.} = \frac{3300 \times R. \cos \phi}{R}$$

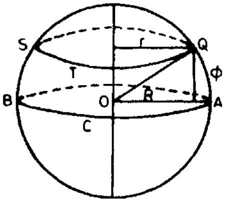

Fig. 1

Note : ABC is terrestrial equator (Fig. 1)

QST is rectified circumference at a place latitude ϕ

$$\frac{QST}{ABC} = \frac{2\,\Upsilon}{2\,R} = \frac{\text{Kojyā}\ \phi}{R}$$

$$\therefore\ QST = \frac{(ABC)\ \text{Kojyā}\ \phi}{R} = \frac{3300.R.\text{Cos}\ \phi}{R}$$

(Refer : Arka Somayāji *Siddhānta Śiromaṇi,* pp. 85-86.)

30. Then dividing it by 60, multiply by 360 and divide by 3300 (*yojanas*). The result gives the difference in degrees of two places i.e. the place of Lanka meridian and that at the *svadeśa*. The method to get the difference in *yojanas* is now given.

31-32a. For these places which have this north-south line as the same (i.e. on the same circle of latitude), the distance in *yojanas* between them is one *nātika* of the rectified circumference (i.e. one sixtieth of it commentator Śankara vārniar) This is the time obtained from *deśāntara.*

32b. 33a. Or else the longitudinal difference in time between any place and that on the equator (can be obtained by noting the difference in the times of the obscuring of the Moon or its coming out (during an eclipse), at these two places).

(Note. Siddhānta Śirōmaṇi Bhūparidhyadyāya : ślokas 4, 5 and 6 explain this process. *Siddhānta Śiromaṇi,* Arka Somayāji, p. 90-91).

Positive and Negative Nature of longitudinal time.

22b. At the place which is to the east of the primary meridian, the planet is always seen earlier and on the western side of it (it is seen) later.

34. The mean position of the celestial bodies in minutes is multiplied by the longitudinal difference in *ghatis* and divided by 60. The result is substracted if the meridian line is eastern and added if western, (to the standard meridian).

The Zero-positions of Planets at the beginning of Kali

35-38a The Zero-position of the Moon in seconds etc. is 4° 45', 46' The mean position of the Moon's apogee is equal to 3^r 29° 17' 5"; of Mars is 11^r 17° 47' in minutes etc.,; For Mercury (and all the previous ones) 36' are to be substracted. For Jupiter 12° 10' is to be added; For Venus one *rāśi*, 6 degrees and 13 minutes are to be added. For Saturn 11^r 17° 20' (is to be added); In the case of the node of the Moon the zero-value 6^r 2° 20' is to be added to the result obtained by subtracting the calculated value from 360° (*maṇḍala*).

Zero-positions of the planets at the Ninth minor yuga

38b-40 These are the zero-positions at the beginning of the Kali yuga. These when added with the amount traversed by each planet in a *yuga* (to be defined below) will give the *dhṛuva* at the beginning of that minor *yuga*. The (minor) *yuga* now being defined is one of 576 years. The distance traversed in this *yuga* is obtained by dividing the number of revolutions (mentioned earlier) by 750.

This distance traversed in a *yuga* multiplied by 8, is to be added to the Zero-positions at the beginning of *Kaliyuga* (to obtain the Zero-positions at the beginning of minor yuga.

(Note : The epoch we obtain corresponds to 4609 of Kali Year or 1507 AD)

The Apogees of planets.

40. The apogees of Mars and other planets (Mercury, Jupiter, Venus and Saturn) are respectively 127°, 220°, 172°, 80° and 240'. Of the sun it is 78'.

Thus in *Tantrasaṃgraha* the first chapter entitled *Madhyamaprakaraṇa* (The mean Planets) ends.

Chapter II
Sphuta Prakaraṇam. The True Planets Anomaly and order of the quadrants

1. The longitude of the planet diminished by its *ucca* is the *Kendra* (anomaly). In that a quarter is equal to three *rāsis* (i.e. 90°). In the odd quadrants, that which is gone (passed over) and yet to go are called the *bhuja (arm)* and *koṭi* (the vertical side). In the even quadrants it is otherwise.

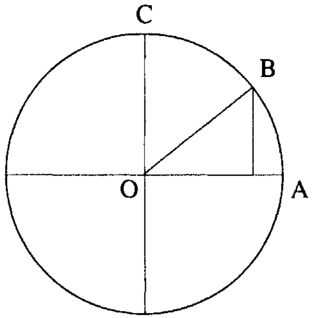

Fig. 2

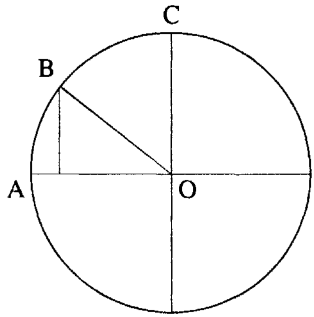

Fig. 3

Note : For Fig. 2, AB = *Bhuja,* BC = *Koṭi.*
For Fig. 3, AB = *Bhuja* BC = *Koṭi*

2-3a. The number of seconds is divided by 225, (The quotient) gives the number of *jyās* that have gone. Again multiply by the remainder, the difference between the R Sine values that is yet to go and that is gone. Divide by 225. Add the result (to the R Sine that is gone). The *bhuja* and the *koṭi* are to be calculated thus. From the R Sine value the arc is obtained by the reverse process.

Note : In ancient Hindu Trigonometry 90° is divided by 24 and the R Sines at an interval of 225' are taken. If $\alpha = 225'$, we have R Sin α, R Sin 2α, R Sin 3α .... and finally R Sin 24α = R sin 90° = R. If any angle is given, say, x = 925', its

R Sine is found in the following manner.

$$R \text{ Sin } 925' = R \text{ Sin } 4\alpha + \frac{25\,(R \text{ Sin } 5\alpha - R \text{ Sin } 4\alpha)}{225}$$

To find the angle, given the R Sine the reverse process is to be followed :

Let R Sin θ = x' Now x' is given. Let the greatest.

R Sine that could be subtracted from it be, Y = R Sin θ'

Let x-y = r. Let D = (R Sin θ – r Sin θ')

Then $\theta = \theta' . \dfrac{r \times 225}{D}$

(Spasṭadhikāra verses 10, 11 *Siddhānta Śirōmani* also give the same method).

To find the R Sine of an arc intermediate between two R Sines with better accuracy

3b. The R Sine of the arc in minutes which is one-eighth of a

Rāśi i.e. $\left(\dfrac{30\times60'}{8} = 225'\right)$ is deficient by 10" from it.

Note : (R Sin 225' = 225' - 10" = 224' 50").

4. Dividing the first R sine by 233 ½ and diminishing the result from the same, the second difference of the R Sines is obtained. That added to it (the first *Jyā)* is the second *jyā.*

Note : R Sin α, R Sin 2α, R Sin 3α ..... are the first, second, third (*Piṇḍa) jyās.* (R Sin α - 0)

(R Sin 2α - R Sin α), (R Sin 3α - R Sin 2α) .....

are the first, second, third .... *Khaṇḍa jyās* i.e. Δ_1, Δ_2, Δ_3,

etc., Hence *Śloka* gives $\Delta_2 = R \text{ Sin } \alpha - \dfrac{R \text{ Sin } \alpha}{233\frac{1}{2}}$

$\therefore R \text{ Sin } 2\alpha = R \text{ Sin } \alpha + \Delta_2 = 448' \; 46''$

5. 6a. The divide by the same divisor (the second R Sine) and subtract the result from the second *khaṇḍa jyā,* to obtain the third *khaṇḍa jyā.* The third *jyā* is got by additing to the second *jyā.* Then from that fourth *jyā* and other *jyās* are obtained in order.

Note : We find that $\dfrac{R \text{ Sin } 2\alpha}{233\frac{1}{2}} = 1' \; 54'' = - \; 1' \; 54''$

Hence $R \text{ Sin } 3\alpha = R \text{ Sin } 2\alpha + \Delta_3$

$$= \text{R Sin } 2\alpha + \text{R Sin } 2\alpha - \text{R Sin } \alpha - \frac{\text{R Sin } 2\alpha}{233\frac{1}{2}}$$

$$= 671' \ 16''$$

(Refer : *Indian Journal of History of Science,* **18** (1), May 1983, pp. 79-81 for further information.)

An alternate method for finding R Sines

6b-7a. Or else first obtain the value of half the diameter (R) and then calculate the other R Sine values. The value in seconds of a circle (360 × 60) is multiplied by 113 and divided by 354,

is the diameter, $\left\{ \text{i.e. } D = \frac{360 \times 60 \times 113}{354} \right\}$

7b. The square root of the difference between the squares of that Radius and the first *jyā* is the last but one *jyā*.

(*Note :* $\text{R Sin } 23\alpha = \text{R Sin } (90{-}\alpha) = \sqrt{R^2 - (\text{R Sin } \alpha)^2}$)

8. Twice difference between the ultimate and the penultimate R Sines is the multiplier. The divisor is half the diameter. (The multiplier and divisor) of the First R Sine is the first difference of the first and second *khaṇḍajyās*.

Note : Here $\Delta_1 - \Delta_2 = 2 \frac{(R - \text{R Sin } 23\alpha)}{R} \text{ R Sin } \alpha$

It could be seen that the above is equivalent to 2 Sin α – Sin 2α = 2(1 – Cos α) Sin α.

9. 10a. With the same multiplicant and divisor, the Second R Sine and the rest in order (are operated upon). Thus the difference between a particular *khaṇḍajyā* and the one that follows it, is found.

So also are the values of the other R Sines (*Piṇḍajyās*). Thus all the R Sines with their fractions are to be obtained properly and read in order.

Note : $\Delta_2 - \Delta_3 = 2 \frac{(R - \text{R Sin } 23\alpha)}{R} \cdot \text{R Sin } 2\alpha$

$$\Delta_3 - \Delta_4 = 2 \frac{(R - \text{R Sin } 23a)}{R} \cdot \text{R Sin } 3\alpha$$

By the method given in *ślokas* (3b-6a), we find that R Sin α = 224' 50" 22''' R Sin 2α = 449' and R Sin 3α = 671' Using 6b-9, we shall find R Sin 2α and R Sin 3α

$$R \text{ Sin } \alpha = 224'\ 50''\ R = \frac{360 \times 60 \times 113}{354 \times 2}$$

$\therefore$ R Sin 2α = 448'46", and similarly R Sin 3a = 670' 48"

To find the R Sine of any desired arc

10-11a. The arc for which the R Sine and R Cosine are required is the desired arc. Find the two R Sines that are close to it, (either by excess or defect), keep these two separate. Find the arc length that is either less or more (than which is nearest to the desired arc).

Note : let θ be the desired arc. If α = 225' find k such that $K\alpha < \theta < (K+1)\alpha$. θ will be nearer either to $K\alpha$ or $(K+1)\alpha$. Find by twice that arc difference $\delta\theta$.

11b-12a. 13751 divided by twice that arc difference in seconds is kept separately as, the divisor D $\left(D = \frac{13751}{2d\theta} \right)$ · for the *bhuja and koṭi* (that were calculated and kept separate earlier) and for mutually obtaining their values correctly.

Note : 13751 is equal to 4 R. Hence $D = \frac{2\text{ R}}{\delta\theta}$

12b. At first divide one of them (*bhuja or koṭi*) and either add or subtract the result from the other ratio (*koṭi* or *bhuja*) according as the arc difference is either more or less.

Note : If $\theta = k\alpha + \delta\theta$, then we find, $\left(\frac{\text{R Sin } k\alpha}{D} + \text{R Cos } k\alpha \right)$

for finding R Sin θ. For R Cos θ we should get

$$\left(\frac{\text{R Cos } k\alpha}{D\ .} + \text{R Sin } k\alpha \right)$$

13-14a. That result is doubled and is operted as before, for obtaining the correct value. Thus calculated the two ratios of the arc are correctly obtained. (Or else) by obtaining the *bhuja* or *koṭi* related to the lesser arc, the other (*koṭi* or *bhuja*) is obtained by taking the square root of the result by deducting its square from the square of *Trijyā* (R).

Note : It $\theta = k\alpha + \delta\theta$ the following results are given.

$$R \text{ Sin } \theta = R \text{ Sin } k\alpha + \frac{2}{D}\left(\frac{R \text{ Sin } k\alpha}{D} + R \text{ Cos } k\alpha\right)$$

$$R \text{ Cos } \theta = R \text{ Cos } k\alpha + \frac{2}{D}\left(\frac{R \text{ Cos } k\alpha}{D} + R \text{ Sin } k\right) \text{Where}$$

$$D = \frac{2\,R}{\delta\theta}$$

To compute the arc given its R Sine according to Mādhava

14-15a. Of the two R Sines (that which is given and) that which is nearest to the given, their difference (is taken). The sum of their complements (*koṭi*) is divided by this difference.

That is the divisor $D = \left(\dfrac{R \text{ Cos } (\theta + \delta\theta) + R \text{ Cos } \theta}{R \text{ Sin } (\theta + \delta\theta) - R \text{ Sin } \theta}\right)$

Where $\theta + \delta\theta = \alpha$. Twice *Trijyā* when divided by that divisor gives the arc difference i.e. $\delta\theta$

Note : Hence $\delta\theta = \dfrac{2\,R}{D} = 2\,R \left(\dfrac{R\,(\text{Sin } (\theta + \delta\theta) - \text{Sin } \theta)}{R\,(\text{Cos } (\theta + \delta\theta) + \text{Cos } \theta)}\right)$

In modern notation, $\delta\theta = \dfrac{2.2 \text{ Cos}\left(\theta + \dfrac{\delta\theta}{2}\right) \text{ Sin } \dfrac{\delta\theta}{2}}{2 \text{ Cos}\left(\theta + \dfrac{\delta\theta}{2}\right) \text{ Cos } \dfrac{\delta\theta}{2}} = 2\,\dfrac{\delta\theta}{2}$

since $\delta\theta$ is small

15b, c. Thus the finding of R Sine of an arc or finding an arc is to be done according to the method of Mādhava. An alternate rule is also given by him, for those who want to get subtler values.

Rule of 'Jīve paraspara Nyāya' – The rule to find the R Sines of the sum or difference of any two arcs.

16. Multiply the two R Sines mutually by their other *jyās* (i.e. *bhuja* A is multiplied by *koṭi* B and *bhuja* B by *koṭi* A) and divide it by the Radius; their sum or difference becomes (the *jyā* of the sum or difference of the arcs). Or otherwise by obtaining the two square roots of the difference between their own squares and the square of the *lamba.*

Note : Commentator Śankara Vāriar makes clear the word, '*itara jyābhyām*' thus : *ato yogaviyogayogye dve api ardhajye parasparsya nijetara jyābhyām svabhujājyām anyasyā; koṭyā anyabhujājyām svakoṭyā ca gunayet :*

The formula is evidently, $$\text{R Sin (A} \pm \text{B)} = \frac{\text{R. Sin A.R. Cos B}}{\text{R}} \pm \frac{\text{R Cos A R Sin B}}{\text{R}}$$

About *lamba* he remarks '*lambānayanam punarubhayorjivayo : samvargata; trijyayā haraṇena kartavyaṃ* | $\frac{\textit{Lamba} = \text{R Sin A.R. Sin B}}{\text{R}}$, Therefore

'otherwise' means finding square root of $$\frac{(\text{R Sin A})^2 - (\text{R Sin A.R. Sin B})^2}{\text{R}}$$

$$\text{and } (\text{R Sin B})^2 - \frac{(\text{R Sin A.R. Sin B})^2}{\text{R}}$$

To find the arc from the R Sine

17. The cube of the arc that is left over (in excess or defect from that of the tabular arc) is divided by six, and then divided by the square of the radius. The arc reduced by this value, becomes the R Sine of that arc. The value is accurate, if the arc is of small magnitude.

Note : If x be the arc, *śloka* gives $\text{R Sin x} = x - \frac{x3}{6R^2}$

This is equivalent to $\text{Sin x} = x - \frac{x^3}{6}$ When x is small.

To find the R sine of any desired arc.

18. Having obtained the R Sine of the arc that is in excess or defect (of a particular arc of the tabular sine) and the tabular sine, keep them separate.

19. Multiply by the R Cosine (*koṭi*) of the arc in excess or defect, the R Sine of that arc of the table that is nearest to it; multiply also the R Cosine of this tabular arc by the R Sine of the arc that is left over.

20. Divide their sum or difference by R. The result in minutes etc., is the true R Sine of the desired arc.

21a. By the same process the R Cosine is to be calculated in a subtle manner

Note : If the desired R Sine is for arc θ, let 0 = Kα ± x. Then x is the *ūnādhika dhanu,* or *śiṣṭa cāpa.* R Sin Kα is the *paṭitajyā* or *svasamīpajyā.* The *śloka* gives R Sin θ

$$= \frac{\text{R Sin K}\alpha\ .\ \text{R Cos x}}{\text{R}} \pm \frac{\text{R Cos K}\alpha\ .\ \text{R Sin K}}{\text{R}}$$

which is clearly based on Sin (A±B) formula, or the method of '*Jīve-paraspara nyāya*' mentioned in *Śloka* 16.

To determine the true position of the Sun, by calculating the mandaphala and Śīghraphala

21b. Multiply by 3 the R Sine and R Cosine (related to the mean anomaly of the Sun) and divide by 80. The two results are the two *phalas* (*bahuphala* and *koṭiphala* respectively).

22 (Of these two) *Doḥphala* converted into arcs is either added to or subtracted from the mean longitude of the Sun according as the kendra is greater or less than 180 (according as it is in Libra onwards of *Meṣa* onwards). Having done so, it is the true longitude of the Sun for the given time.

23a. (Since longitude thus obtained) is calculated with mean civil day, again it should be corrected for the time of true Sun rise.

Ascensional differences in prāṇas (Fig. 4)

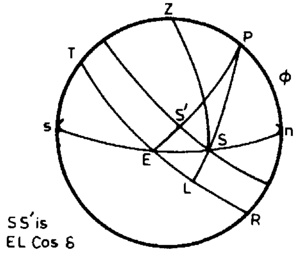

Fig. 4

TR = Equator

S, Ś = Positions of Sun on the Ecliptic

P = Pole

PN = ϕ

SŚ = EL Cos δ

23b. (From the true longitude at rising), the R Sine of the longitude of the Sun in degrees after correcting it for *ayanāṃśa* is to be found.

24. Then it is to be multiplied by the R Sine of 24° and divided by R, that is the true value of the R Sine of the declination of the sun at that instant.

Note : The *śloka* gives, $R \text{ Sin } \delta = \dfrac{R \text{ Sin } \lambda \,.\, R \text{ Sin } 24^\circ}{R}$

Taking $\omega = 24^\circ$ the result follows by applying sine formula for Δ rSD (Fig. 5)

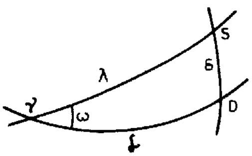

Fig. 5

25a. The square root of the difference of the squares of that and R is *dyujyā* (R Cos δ), its koṭi

25b.26a The square root of the difference between R Sin λ and R Sin δ, or else, the R cosine of the maximum declination (R Cos ω) multiplied by R Sin λ and divided by R gives the *iṣṭa koṭi*.

Note : Hence *Iṣṭa Koṭi* = $\sqrt{R^2 \text{ Sin}^2\lambda - R^2 \text{ Sin}^2\delta}$

$$(\text{or}) = \frac{R \text{ Cos } w \,.\, R \text{ Sin } \lambda}{R}$$

This follows by using value of R Sin δ obtained from *śloka* 24.

Arkabhuja in asus

26b. (The *Iṣṭa Koṭi*) is multiplied by R and divided by *Iṣṭadyujya* (R Cos δ) and the result rendered in arcs is the *bhuja* of the Sun in *prāṇas*.

Note : *Arkabhuja* Mentioned in this *śloka* is nothing but the right ascension of the Sun r D (Fig. 5) The *śloka* gives $\text{Sin } \alpha = \frac{\text{Cos } \omega \,.\, \text{Sin } \lambda}{\text{Cos } \delta}$. This could be proved by using the two results. Sin α = Tan δ Cot ω and Sin δ = Sin λ. Sin ω from Δ rSD. Arkasomayāji in his Śiddhānta Śiromaṇi explains how the ancients derived the above formula

$$R \text{ Sin } \alpha = R. \sqrt{\frac{(R \text{ Sin } \lambda)^2 - (R \text{ Sin } \delta)^2}{(R \text{ Cos } \delta)}} \quad \text{on pages 189-190}$$

17a. The difference between the *arka bhuja* and the longitude in minutes is to be carefully recorded separately.

Note : Śankara Vāriar states that this difference is known as *Prānakalāntara* and that the utility of the same would be declared later on.

27b. 28a. The equinoctial shadow(s) multiplied by R Sin δ, divided by 12 gives the R Sine of earth, that is *kṣitijyā*. This multiplied by R and divided by the desired R Cos δ and rendered into arcs is the *cara* in *asus*.

Note : $Kṣitijyā = \frac{\text{s.R. Sin } \delta}{12} = \frac{12 \text{ Tan } \phi \,.\, \text{Sin } \delta}{12}$

and Sin El = Tan ϕ Tan δ

Now cos h = – Tan ϕ Tan δ

Also Cos (90 + EL) = – Sin EL = – Tan θ, Tan δ

Since SS_1 = EL. Sin (90 – δ) = EL Cos δ

SS_1 = *Kṣitijyā*, (See figure under *śloka*, 23a.)

Sun's daily motion in minutes of arc

28b-30a. If the *dohphala* of the Sun, its cara in *prāṇas* and the *prānantara* in minutes, are of he same sign, they are to be added; if the signs are different, must be subtracted (one from the other)

The result is multiplied by the mean daily motion of the Sun, and divided by 21600 minutes. This result is to be applied to the mean Sun, (the mean Sun at the true Sunrise is thus obtained), Or else (the sum of difference of the three factors) is multiplied by the true daily motion of the Sun, (in minutes) (then divided by 21600) and the result applied to the true Sun at mean Sunrise (for obtaining the true Sun at the true Sunrise at the desired place).

Application of Cara saṃskāra to the true position of planets

30b. When the Sun is in the northern direction, the *caraprāṇa* is to be subtracted. In the southern hemisphere it should be added (for the Sun at rising).

31a. The correction is reversed in the case of the setting Sun. No correction is to be done for the Sun at moon or midnight.

31b. 33ab. The *praṇakalāntara* (*śloka*, 27a), is positive or negative according as the *sāyaṇaravi* (longitude of Sun + *Ayanāṃśa*) is in even or odd quadrants. The dohphala (of the Sun) is calculated as explained earlier. By the value obtained from these (three mentioned in *śloka* 286-30a) corresponding to the Sun, the mean or true daily motion of the planets is multiplied and divided y 21,600. This must be added or subtracted as stated earlier. When the planet has retrograde motion reverse should be the process for finding (the mean or the true position of) the planet at the true Sunrise.

Measure of day and night after applying cara saṃskāra

33b. 34. When the Sun is in the northern hemisphere, add the *caraprāṇa* to one-fourth of a day and night (i.e. for 15 *ghaṭis*). It the Sun is in the southern hemisphere it should be subtracted; the result is half duration of day. The reverse is to be done for getting half-duration of night. These multiplied by two give the duration of the day and night. For the Moon etc., with their own cara in *asus* (one can find out the duration between any two successive rising of the Moon etc., at the horizon of the given place).

To find the true position of the Moon

35. In case of the Moon and its apogee, after obtaining the *cara* etc., of the Sun, the correction for the mean position of the Moon is to be done (as described earlier to get the mean position of the Moon and its apogee at the true sun rise).

36. Both the doḥphala rendered in arcs is to be applied to the Moon position for obtaining the true position.

Converting into arcs of the cara and jyā

37. By following the rule as stated in '*śiṣṭa cāpa gaṇa*' etc., (II. 17), find the

difference between the R Sine (*jyā*) and its arc (*cāpa*). By adding this to the given R Sine, the method is to be followed successively. Or else it could be obtained from the Tabular R Sines.

Note : The next śloka gives the method from Tabular R Sines

38. 118, 18, 103 is equal to the square of R. It is multiplied by one, two etc., divided by 10 and its cube root (is taken).

39. The result in seconds is to be added to the first, second.... R. Sines.

In the case of *cara* and the doḥphala, similarly the arc that is small is to be obtained.

Note : $\left(\frac{R^2}{10}\right)^{1/3}, \left(\frac{2R^2}{10}\right)^{1/3}$ ... are given to be equal to

$(\alpha + \delta\theta) - \text{Sin}\,(\alpha + \delta\theta), (2\alpha + \delta\theta) - \text{Sin}\,(2\alpha + \delta\theta)...$ etc

To find the hypotenuse related to Mandocca and Śīghrocca.

40, 41a. Add the *koṭiphala* to R in the first and fourth quadrants; deduct it from R in the other quadrants. The hypotenuse is the square root of the sum of this square and that of *bhuja phala.* This is the hypotenuse (*karṇa).* In *Maṅda* process method of iteration is to be followed. For the *Śīgra* process, it is not to be followed.

Karṇa = $\sqrt{(R \pm \textit{koṭiphala})^2 + (\textit{bhujaphala})^2}$

Note : Mandakarṇa (fig. 6)

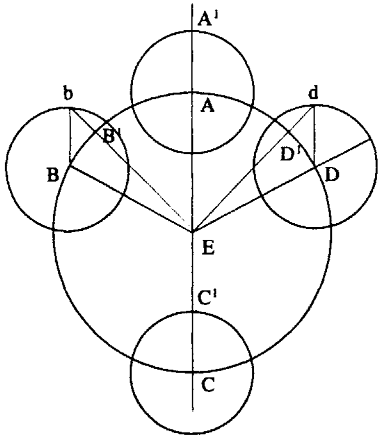

Fig. 6

E Earth's Centre

AB' B-Mean orbit, *Kakṣā Maṇḍala*

Circle B, *Nīcoccavṛtta*

A' is *Mandocca*

AÊB = *Mandakendra*

b is the true planet and appears to be at B'

BB', is the *Mandaphala* or equation of Centre, so also is DD'

Eb *Mandakarṇa.*

(*A Critical study of the Ancient Hindu Astronomy,* D.A. Somayāji. p. 74-75).

41. 42. The hypotenuse multipled by the *bhujaphala* and *koṭiphala* separately, is to be divided by R (The values obtained are the new *bhuja* and *koṭiphalas*). From these values of *bhujaphala* and *koṭiphala,* the hypotenuse is to be obtained once again. This *karṇa* is to be multiplied by the previous value of *bhuja* and *Koṭi phalas* and divided by R. From the value of each such hypotenuse, the method of successive approximation is to be carried out.

Note : $$(Doḥphala)_2 = \frac{(Doḥphala)_1 \times \text{Karṇa})}{R}$$

$$(Koṭi\ phala)_2 = \frac{(Koṭiphala)_1 \times \text{Karṇa}_1}{R}$$

$$\text{Now } (Karṇa)_2 = \sqrt{(R \pm Doḥphala_2)^2 + (Koṭiphala_2)^2}$$

To find Mandakarṇa, an alternate method without successive approximation

43. The square root of the difference of the squares of R and the *manda dohphala* (is taken). from this is subtracted or added the *koṭiphala* according as the (mean anomaly lies within 6 *rāsis* beginning) in *Makara* or *Karkataka.* This indeed becomes what is called the *viparīta karṇa* (V.K.)

44. The square of R Divided by that (V.K.) is the *Aviśeṣa karṇa* or hypotenuse obtained without the effort involved in carrying out the method of successive approximation. Thus by an alternate method, the *manda* hypotenuse is to be obtained at one step as per the method enunciated by Mādhava.

To find the true Sun from Mandakarṇa :

45. The R sine of the dohphala is multiplied by R and is to be divided by this hypotenuse. This is the true R Sine. The arc applied to the mandocca or *nīcca* correctly will give the true *ucca* or *nīcca.*

Note : The Commentator explains the import of the word *yuktita* as follows :

In the first quadrant, the arc *s* is to be added to the *mandocca,* in the second quadrant (180° – *s*) is to be added to the Mandocca;, in the third quadrant *s* is added to the lowest *(svanīce)* and in the fourth quadrant (180° – *s*) is added to the lowest.

Mean position from the True Sun

46-47. To obtain mean Sun from the true sun, *ucca* is subtracted from the true

Sun. After calculating the *bhujaguṇa* and the *koṭiguṇa*, depending upon, if the *kendra* lies within the six signs beginning with *Makara* or *Karkataka*, the *antyaphala* (radius of the epicycle) is subtracted from or added to the *bhuja*. That which is the square root (of the above sum) is the hypotenuse, that is termed *Viparīta karṇa*. *That* hypotenuse divided by the square of R is the true hypotenuse *(pratimaṇḍala karṇa sphuṭah:)*.

48-49. (Then multiply the true hypotenuse by *bhujajyā* and divide) by the radius. The result converted into arcs is added to the sun's apogee (if the *kendra* lies) in the first quadrant. If in the second quadrant, the result substracted from 180°, if in the second quadrant, the result substracted from 180°, if in the fourth quadrant the result subtracted from the circle i.e. 360° (is to be added). When the corrections are done, thus, the mean position becomes more accurate.

Of the two, *koṭijya* and *antyaphala*, that which is more is (to be taken as) in the first quadrant. In the fourth...

50b. Everywhere the *karṇa* (hypotenuse) is taken equal to the radius; and the *viparītakarṇa* is for the radius R.

Commentator : For a particular *Viparītakarṇa, mandakarṇa* is R, then for V.K. equal to R, what is the *mandakarṇa?* It is $\frac{R}{V.K.} \times R$. Hence the precess of finding *Mandakarṇa* is described as dividing the square of R by the *Viparīta karṇa.*

Alternate method for finding the mean planet from True Sun

51.a. From the true positions of the Sun and Moon subtract their *mandoccas*. Find the *doḥphala* and *koṭiphala* from the remainder. Find the *karṇa* once (as per the rule depending upon if the *Kendra* lies within the 6 signs beginning with) *Karkaṭa* or *Mṛga*.

Commentator : If the mean position is in *Mṛga* substract the *koṭiphala* from R. In Karkaṭa add it to R. The square of R thus acted upon and the square of the *doḥphala* are added. The square root is *Viparīta karṇa.*

Hence *Viparīta karṇa* = $\sqrt{(R \pm Koṭiphala)^2 + (doḥphala)^2}$

51b. The product of doḥphala and R in divided by the hypotenuse and the result rendered into arcs. Add it to (or subtract it from the true position depending upon if the *Kendra* lies within 6 signs beginning with *Meṣa/Tula* respectively). This is done for obtaining the mean position (of the Sun and the Moon).

Alternate method for mandakarṇa.

52. From the mean and true positions (of the Sun or Moon) substract their own *mandocca*. Obtain the two *bhuja jayā*. The first (*doḥphala* obtained from the mean position) of the two, is multiplied by R; The result is divided by the other (the *doḥphala* obtained from the true position). The result is the true value of the hypotenuse, (*sphuṭa mandakarṇa*).

$$\textit{Mandakarṇa}\ K = \frac{(\text{Mean position-}\textit{Mandocca})}{(\text{True position-}\textit{Mandocca})} \times R$$

To find the instantaneous velocity of Sun and Moon

53. Divide the product of the daily motion of the Moon and its *koṭiphala* in minutes, by the square root of the difference of the squares of R and Moon's *bāhuphala*. The result which is thus obtained is subtracted or added (depending upon if the motion is in the *rāśis* beginning with Capricon or Karkaṭa). That becomes the true motion of the Moon. The true of the Sun at any moment is also obtained similarly.

Note : Bibhutibhusan Daṭṭa and Awadesh Nārayan Singh have referred to these verses 53-54 in their article 'Use of Calculus in Hindu Mathematics' published in *Indian Journal of History of Science,* **19** (2), April 1984 :

On page 100 they write ... Nīlakṇṭha has made use of a result involving the differential of an inverse sine function. This result expressed in modern notation, is

$$\delta\{\text{Sin}^{-1}(e \sin\omega)\} = \frac{e \cos w \, d\omega}{\sqrt{1-e^2 \sin^2\omega}}$$

(e = eccentricity or the sine of the greatest equation of the orbit)

True asterism at desired time

55. 56a. The true longitude of the lord of the night (Moon at Sunrise) is converted into minutes and divided by 800. The result (quotient) will be the star (asterism) that has elapsed since *Asvini.*

The balance (remainder) that should elapse or has elapsed, multipled by 60 gives the *Nāḍis* that are yet to go, or gone and divided by the true motion (of the moon in minutes) at Sunrise.

True Lunar Day at the desired time (Tithi)

56b. 57a. Diminish the true longitude of the Sun from that of the Moon, convert the result into minutes. Divide by 720. The quotient gives the number of lunar days that have gone (counting) from the new Moon during the bright fortnight, *(śukla pakṣa).*

Karṇa at the desired time

57b. The reminder after division and the balance obtained by diminishing the same from the divisor are multiplies by 60 and divided by the difference in minutes of their true daily motions.

The results are the values in *nādis* that are gone yet to go (in the *tithi* or the desired time).

58. (The difference between the positions of Moon and Sun in minutes) is divided by half the divisor for *tithi* (i.e. 360). The result is the *karṇa* counted from Baba etc.,

Commentor : The *nādis* elapsed and yet to elapse in the *karṇa* are to be calculated as stated earlier in 57b).

59a. In the bright fortnight of the moon the *karṇas* are without form and in the darker fortnight with form.

Commentator : Virūpa are Baba, *Bālava* ... etc., *Sarūpa* are Lion, Tiger... etc.

The Yoga at the desired moment

59. The *Yogas* starting from *Viṣhkambha* etc., are obtained by adding the true positions of the Sun and Moon and dividing the sum by 800. The remainder after division, and the balance obtained by diminishing it from the divisor are divided by the sum of the true daily motion in minutes of the Sun and Moon. The results give the *nādis* that have gone or yet to go (in that *Yoga*).

True position of Mars etc.,

60. The method of finding the true positions of Mars, Jupiter and Saturn are given by the previous wise *ācāryās* (like Āryabhaṭa) as applying the four rules in the following order, *viz.,* first *manda* related process, then *śīghra* related process, once again *manda* and finally the *śīghra.*

The compute the true position

61a. One-eighth of the *doḥphala* and *koṭiphala* in the case of Saturn diminished by their own 40th part, are the true *dohphala* and *koṭiphala*

$$\left(\left(\frac{1}{8} - \frac{1}{320}\right) \text{Sine/Cos } kendra\right)$$

61b–62a. The *mandaphala* of Mars and Jupiter (are as follows). Their *dorjyā* is divided by R and to the result seven is added; that is the multiplier for both *doḥphala* and *koṭiphala* and the divisors are 39 and 82 for Mars and Jupiter, for obtaining the *doḥphala* and *koṭiphala* in the *manda* process.

62b. To the mean longitude kept separate, half of the arc associated with *doḥphala* is to be added or subtracted.

63a. 64. From the longitude of the planet thus obtained the mean longitude of Sun is subtracted and the *bāhu* and *koṭi* are obtained. the *bāhuphala* is divided by R for Jupiter and Saturn and then subtracted from 16 and 9 respectively. For Mars its own *bāhujyā* is divided by R, and the result is doubled and then subtracted from 53. These results are the multipliers.

65. The divisor for all of them is 80 (and leads to their *bāhuphala* and *koṭiphala*). Obtain the *karṇa* (hypotenuse) only once as explained earlier.

66. The doḥphala is multiplies by R and divided by this hypotenuse. That which is the arc (of this result) is the *śīghra phala.* Half of this value is added or subtracted to the mean planet corrected by the *manda* correction, explained earlier.

67a. From the result, subtract the *mandocca* and obtain the *manda phala* and apply it wholly to the original mean planet.

67b. From the result thus obtained, preserving it separately the *śīghraphala* is obtained as earlier and is expressed in arcs.

68a. The whole value (is added to or subtracted from) the *manda* corrected planet obtained in the third stage. That value then becomes the true planet.

68b. From the mean position of Mercury, subtract the value of its own *mandocca* and the two values, the *doḥrjyā* and *koṭjyā* (are obtained).

69. From one-sixth values of these two, the *karṇa* (hypotenuse) is to be found by successive iteration. The *doḥphala* only (converted into arcs) is to be added to or subtracted from (the mean value) according as the mean planet lies within 6 signs beginning with *Tulā* (*Juka,* Libra) or *Meṣa* (*Kriya,* Aries).

70. The value thus obtained operating with the mean gives the true position of

Budha. Then the mean position of the Sun is to be subtracted (and the *śīghra kendra* is obtained). From that the *bhuja* and *koṭi* are calculated.

71. The R sine of the bhuja *(doḥrjyā)* is multiplies by two and divided by R. The result is subtracted from thirty-one. (The result is) the multiplier. That multiplied by the *manda-karṇa* (calculated iteratively) and divided by R is the true multiplier.

72. The R Sines of the *bahu* and *koṭi* multiplied (by the above multiplier) are divided by 80. The two results (are the true *bahu* and *koṭiphala* in the *śīghra* process. The *doḥphala* multiplied by R is divided by this *Karṇa* (This is the true *śīghraphala*).

73. The result converted into arcs is fully added to or subtracted from the mean longitude of the Sun according as it lies within the first 6 signs beginning with Aries (*Meṣa*) or Libra (*Tulā*).

74. Thus the mean position of the Sun corrected by the *śīghraphala* of Mercury, gives the true position of Mercury. The true position of Venus is also found similarly.

Speciality in the case of Venus

75. The 240th part of the R Sine of the *mandakendra* is added to 14. That is the divisor (for obtaining *doḥphala* and *koṭiphala*). The *bāhu* and *koṭijyās* divided by this divisor are the *doḥphala* and *koṭiphala* in the *manda* process.

76. Havin, applied the arc corresponding to the *doḥphala* to the *madhyama,* let the *śīghra* correction and the *aviśeṣa karṇa* be carried out.

77-78. The *doḥrjyā* (associated with the *śīghra kendra* of Śukra) is doubled divided by R. It is subtracted from sixty diminished by one (i.e. 59). That is the multiplier. As done earlier the true value is to be found using the *manda karṇa.* The multiplier multiplied by *mandakarna* and divided by R is the true multiplier. The *bhuja* and *koṭijyās* (associated with the *śīghra kendra* of Śukra) are multiplied by this multiplier and then divided by 80. The results are the *śīghra doḥphala* and *koṭiphala* of Venus.

79. The *doḥphala* is multiplied by R and divided by the *śīghrakarṇa.* The result in arcs is applied to the mean position of Sun. That is the true position of Venus.

Computation of the daily motion of the planets

80. (The difference in the true positions on any day and the following day is the daily motion on that day). If the true longitude on the following day is less than the longitude of a day, then the difference gives the amount of retrograde motion, otherwise the differences gives the true daily motions of the planets.