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THREE OLD INDIAN VALUES OF π

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Three values of π from the *Śatapatha Brāhmana and Baudhayāna Śulbasūtra* are discussed. These values emerge when squares are transformed into circles of equal area, a commoly occurring operation in Vedic altar construction.

Keywords : Ancient geometry, π

INTRODUCTION

In this article we describe three hitherto neglected references to π, one from *Śatapatha Brāhmaṇa* (ŚB) and the others from *Baudhāyana Śulbasūtra* (BŚS). These references relate to construction of altars of certain shapes and sizes the background to which is described in the analysis of Vedic geometry by Seidenberg[1]. Histories of Indian mathematics generally begin with the geometry of the Śulbasūtras but Seidenberg showed that the essentials of this geometry were contained in the altar constructions described in the much older *Śatapatha Brāhmaṇa* and *Taittirīya Saṃhitā.* More recently it has been shown that Vedic astronomy goes back to at least the third millennium BC[2] and so a concomitant mathematics and geometry must have existed then. It has also been shown that an astronomy is coded in the organization of the Vedic books itself.[3]

Meanwhile archaeological discoveries have had major implications for the understanding of ancient Indian chronology.[4] Briefly, discoveries related to the drying up around 1900 BC of the Sarasvati river, the prominent river of the Ṛgvedic age, indicate that this epoch must be considered the *final limiting point in time* for Ṛgveda. Therefore, it seems reasonable to assign second millennium BC for the Brāhmaṇa literature as is also attested by their internal astronomical evidence[5]. The Śulbasūtras have been traditionally dated to later than 800 BC, but Seidenberg suggests that their knowledge belongs to a much earlier period.

Elsewhere, I have argued for a value of π implicit in the organization of the Ṛgveda and this should be earlier than the age of the Brāhmaṇa literature. But here we are

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concerned only with early explicit values of π. The ŚB is not mentioned as a text in the list of values of π in the review by Hayashi et al[6], who have missed the antecedents of the history of π in India. We also show a connection between the approximations of ŚB and BŚS.

TRANSFORMING A SQUARE INTO A CIRCLE

The transformation of a square altar into a circular one is a commonly occuring theme in the altar ritual. This leads to many geometric and algebraic results and also many values for π.

ŚB 7.1.1.118-31 describes the construction of a circular *gārhapatya* altar starting with bricks of different kinds.

> *sa catasraḥ prācīrūpadadhāti dve paścāt tiraścyau dve purastāt. tad yāścatasraḥ prācīrūpadadhāti sa ātmā. tad yat tāścatasro bhavanti caturvidho hyayamātmā 'tha ye paścāt te sakthyau ye purastāt tau bāhū yatra vā 'ātmā tadeva śiraḥ. [ŚB 7.1.1.18]*
>
> "He puts on (the circular site) four (bricks running eastwards; two behind running crosswise (from south to north), and two (such) in front. Now the four which he puts on running eastwards are the body; and as to there being four of these, it is because this body (of ours) consists of four parts. The two at the back then are the thighs; and the two in front the arms; and where the body is that (includes) the head". [Eggeling's translation]

A figure detailing this construction (part 3, page 302) is given by Eggeling[7]. Elsewhere, as in ŚB 10.2.3.1-3, there is unmistakable mention of a *āhavanīya* altar of one *vyāma* square. The relevant passage from ŚB 10.2.3.3 is:

> [The space of a] *vyāma* which was (marked off), is the womb of the *gārhapatya*, for it was that womb that the gods beget the *gārhapatya*; and from the *gārhapatya* the *āhavanīya*.

Later texts such as the Śulbasūtras explicitly speak of the gārhapatya altar being equal to one *vyāyāma* measure, in either the square or the circular form (see e.g. BŚS 7.4-5). Seidenberg analyzed these constructions in his papers and concluded that the *gārhapatya* and the *āhavanīya* altars, generally circular and square respectively, were of equal area. But he added that "one will not come, without interpretation, to an unambiguous meaning from such passages"[8].

For ready reference we provide the relevant units of measurement:

1 *puruṣa* = 1 *vyāma* = 5 *aratnis* = 120 *aṅgulas*

We argue here that ŚB, in fact, does provide conclusive evidence of the identity of the areas of the two altars. The construction of the *gārhapatya* altar is using oblong and square bricks (Figure 1). The bricks come in a variety of sizes and the question is to determine whether the context can fix the specific size meant in ŚB.

The square bricks used are either one-fourth, one-fifth, one-sixth, or one-tenth of a *puruṣa* on each side[9]. From the nature of the construction the only square brick of the types that will fit into the scheme is the one-fifth, the *pañcami*, which is 24 x 24 square *aṅgulas* which is the same as 1 x1 square *aratni*. The other bricks lead to area that is either too large or much too small to fit our equal area requirement. The oblong brick is therefore 48 x 24 square *aṅgulas*.

Since the inscribed square in Fig. 1 is 4/5 x 4/5 square *puruṣa*, we see that the diameter of the circular *gārhapatya* altar is $\frac{\sqrt{32}}{5}$ *puruṣa*. Since its area is taken to be equal to that of the one square *puruṣa āhavanīya* altar, this leads to

$$\frac{\pi \times 32/25}{4} = 1 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (1)$$

From this it follows that

$$\pi_1 = \frac{25}{8} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (2)$$

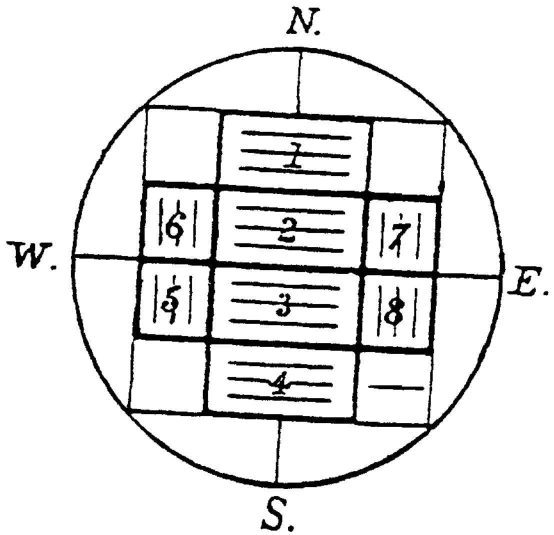

Fig. 1.

CONSTRUCTION FROM BŚS

Sen and Bag[10] have described several approximations of π in the Śulbasūtras. But here we are intersted in BŚS 16.6-11, which gives a construction for converting a square into a chariot-wheel or a wheel with spokes (*rathacakra*). As we will see this starts with a very specific design using bricks of a certain size, but ultimately the design is described in bricks of entirely different shapes with circular arcs.

In this design one starts with an area equal to 225 square bricks; this is now augmented by 64 more bricks so that one has a new square equal to $17^2 = 289$ bricks. Each brick is to be of an area $\frac{1}{30}$ square *puruṣa*.

tāsāṃ dve śate pañcaviṃśatiśca sāratniprādeśaḥ saptavidhaḥ saṃpadyate (16.8)
tāsvanyāścatuḥṣaṣṭimāvapet. tābhiḥ samacaturaśraṃ karoti.
tasya ṣoḍaśeṣṭakā pārśvamānī bhavati.
trayastriṃśadatiśiṣyante tābhirantānsarvaśaḥ paricinuyāt. (16.9)
nābhiḥ ṣoḍaśa madhyamāḥ.
catuḥṣaṣṭirarāścatuḥṣaṣṭirvediḥ nemiḥ śeṣāḥ (16.10)

With 225 of them [bricks] is produced the seven-fold [altar] with two aratnis and [one] prādeśa. (16.8)

To these [225] another 64 [bricks] are added and with them a square is made. (At first) a square is made with a side containing 16 bricks, leaving a balance of 33 bricks. These are placed on all sides. (16.9).

16 (bricks) at the centre constitute the nave; 64 (bricks, thereafter) constitute the spokes and 64 the empty spaces (between the spokes); the remaining (bricks) form the felly. (16.10)

The reference in the second part of 16.8 is to the area of the altar which is to be $7\frac{1}{2}$ square *puruṣa* as 2 *aratnis* and one *prādeśa* equal half a *puruṣa*. The main fire altars in the *agnicayana* ritual were to be of this size.

The construction that follows begins with a square of 225 units which is later enhanced to a square of 256 units. The remaining 33 units are distributed around this larger square to give us a circular altar of diameter 17. Sen and Bag have taken this distribution of 33 units to lead to another square of 17 x 17 and that is correct as far as the design goes, but we will show later that there is also the intent make a circular area of diameter 17 units (Fig. 2).

The next sūtra speaks of the further development of the design into a wheel with spokes : the centre (nave), the middle (spokes and spaces), and the outer (felly). Next, BŚS 16.11 speaks of *nemimantataścāntarataśca parilikhya*, or the "outer and the inner enclosing the felly done into circles", but there is no reason to assume that this is not a reiteration of the step carried out in BŚS 16.9.

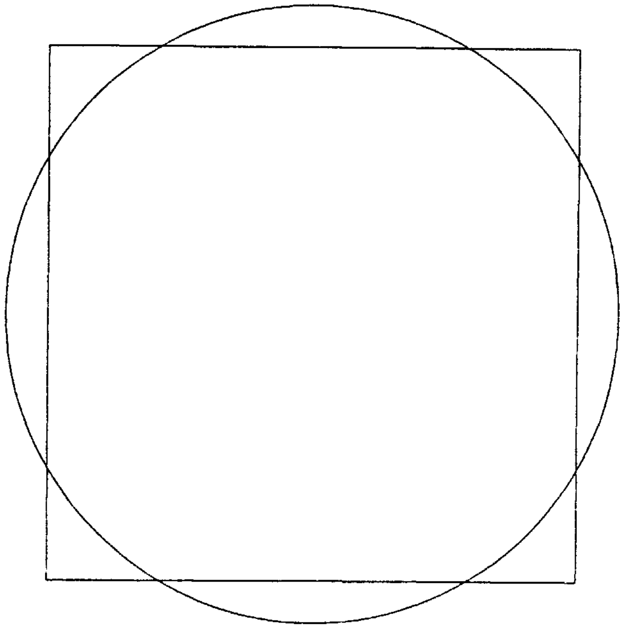

Fig. 2

If we accept for the moment that we have an equality of a square of side 15 units into a circle of diameter 17 units,

$$\pi \times \frac{289}{4} = 225 \quad \text{.......} \quad (3)$$

Or, we obtain the approximation

$$\pi_2 = \frac{900}{289} \quad \text{.......} \quad (4)$$

The important point in this transformation of the square into the circle is that the diameter of the circle is two units greater than the side of the square.

Note that the inner square of 16 bricks becomes the nave and, of the next set of 128 bricks, exactly half go in to the formation of the spaces between the spokes. This means that 64 bricks are left over; these are removed and so the area of the altar remains $7\frac{1}{2}$ square *puruṣa*.

One might object that the transformation of the outer square into the circular felly with diameter of 19 units means that there was no need to convert the first square of 15 x 15 into the circle of diameter 17 units. But, in reality, we do need a circle in the middle of the felly; this is described in the sūtra 16.12:

nemiṃ catuḥṣaṣṭiṃ kṛtvā vyavalikhya madhye parikṛṣet. (16.12)

After dividing the felly into 64 equal parts and drawing lines, a circle is drawn through the middle (of the felly). (16.12)

Although this middle is not specified, it is almost certain that it is the first circle of diameter 17. That is because the separation between these two circles is one brick-width on each side. This strengthens the view that the sūtra 16.9 establishes the method which is used in going from the square of 17 x 17 to a circle of diameter 19. This construction requires first increasing two sides of the original square by one unit, and then distributing the remaining bricks to round off the sides as in Fig. 2.

The outer construction corresponds to a value of π equal to

$$\pi \times \frac{361}{4} = 289. \qquad \text{...........(5)}$$

or,

$$\pi_3 = \frac{1156}{361}. \qquad \text{...........(6)}$$

So we have two further values for π, and it is interesting to note that one of these is very similar to the one in ŚB which can be expressed as $\frac{900}{288}$. It is plausible that the approximation (4) arose from the realization that 1 more than 288 was a square. But the approximation (4) is worse than that of (2). A return to the earlier value is seen in the later *Mānava Śulbasūtra* (MŚS) where the chariot-wheel (*rathacakra*) altar uses 344 bricks instead of the 289 of BŚS. The details of the construction are not clear but it appears that the value of π was $\frac{1075}{344}$. Support for such an interpretation comes from the rule MŚS 11.15 where we encounter precisely the same value[11]. Furthermore, since these texts list a variety of values which, it is clear, were taken to be approximations.

The conversion of squares into circles is the basic geometric issue in BŚS 16, which can be seen from the nature of the Fig. 3 emerging at the conclusion of the construction[12]. Notice that the original bricks are now replaced by new kinds of "bricks" which total 200 as required by the rules of altar design. These new bricks are segments obtained by dividing a circle radially and circularly.

It is significant that the representation of a square of side 15 by the circle of diameter 17, and that of a square of side 17 by the circle of diameter 19, are two of the best three such approximations that can be obtained where the difference in each set of numbers is 2. An even better approximation is to represent a square of side 16 by the circle of diameter 18; but these numbers do not arise in this altar design.

CONCLUDING REMARKS

The altar constructions in the Śulbasūtras use bricks of such great variety that it is likely that in most cases the "ritual" represented just conceptual geometric exercises. This may be seen in Fig. 3 which gives the final arrangement of bricks in the first layer of the chariot-wheel altar. This first layer requires seven different types of bricks and the next layer requires another nine different types. Most of these bricks are not rectangular. It is highly unlikely such bricks were actually made using casts. Ancient ruins show no trace of these bricks.

Śulbasūtra constructions may be seen as defining a discrete geometry. The use of "bricks" implies changes by pre-defined elementary areas.

This note supports the view that the Śulbasūtras represent a continuation of the

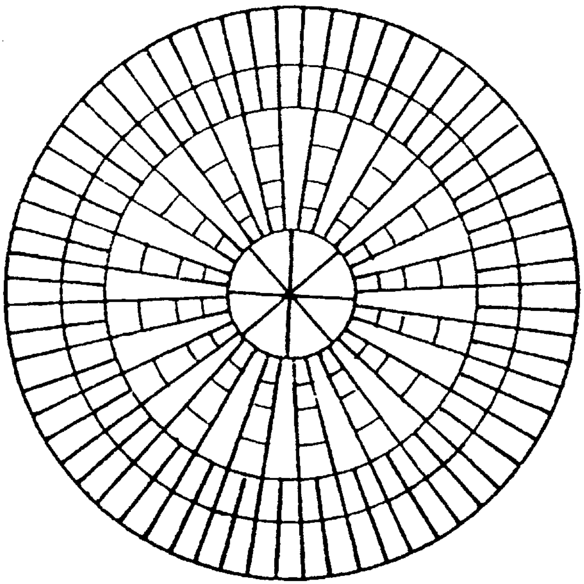

Fig. 3.

geometric tradition of the Brahmaṇas. Furthermore, the relationship between the chariot-wheel constructions of two different texts indicates that many different approximations for π were used.

References and Notes

1. Seidenberg, A. "The Origin of Mathematics," *Archive for History of Exact Sciences*, **18** (1978), 301-342; A. Seidenberg, "The geometry of the Vedic rituals," In *Agni*, F. Staal (ed.), Berkeley, 1983.

2. Frawley, D. "Planets in the Vedic literature," *Indian Journal of History of Science*, **29** (1994), 495-506; Kak, S.C. "Knowledge of the planets in the third millennium BC." *Quarterly Journal of the Royal Astronomical Society*, **37** (1996), 709-715.

3. Kak, S.C. *The Astronomical Code of the Ṛgveda*. Aditya, New Delhi, 1994; see also Kak, S.C. "The astronomy of the age of geometric altars", *Quarterly Journal of the Royal Astronomical Society*, **36** (1995), 385-396.

4. Feuerstein, G. Kak, S.C. Frawley, D. *In Search of the Cradle of Civilization*. Quest Books, Wheaton, 1995.

5. Sengupta, P.C. *Ancient Indian Chronology*. University of Calcutta, Calcutta, 1947.

6. Hayashi, T. Kusuba, T. and Yano, M. "Indian values of π derived from Āryabhaṭa's value," *Historia Scientiarum*, **37** (1989), 1-16.

7. Eggeling, J. (tr.), *The Śatapatha Brāhmaṇa*. Motilal Banarsidass, Delhi, 1988.

8. Seidenberg, A. "The geometry of the Vedic rituals," In *Agni*, F. Staal (ed.), Berkeley, 1983, page 114.

9. Sen S.N. and Bag, A.K. *The Śubasūtras*. Indian National Science Academy, New Delhi, 1983.

10. ibid.

11. This figure, taken from the book by Sen and Bag (P. 218), does not show the circle dividing the felly according to the interpretation in this paper.

12. Gupta, R.C. "New Indian values of π from the Mānava Śulba Sūtra," *Centaurus*, **31** (1988), 114-126.