

MAHALANOBIS AND THE ART AND SCIENCE OF STATISTICS: THE EARLY DAYS

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Prasanta Chandra Mahalanobis's initiation to Statistics was somewhat serendipitous. He taught himself statistics during the early part of this century, and began practising statistics from his own home in Calcutta from 1917. He set up the Statistical Laboratory in the Presidency College, Calcutta, sometime in the twenties. Statistics, in the modern sense, was born in India through the pioneering studies of Mahalanobis. Mahalanobis combined his exceptional flair for empirical studies and decision making, in raising statistics to awesome heights. He also investigated and solved many theoretical problems in statistics, virtually all of which arose from his empirical studies. In additions to carrying out statistical investigations, Mahalanobis played a key role in the organization of statistics in India, by establishing the Indian Statistical Institute (in 1931), the National Sample Survey (in 1950) and 1: Overview the Central Statistical Organization (in 1951).

1: OVERVIEW

Towards the end of the nineteenth century, Karl Pearson, in response to some biometrical questions raised by Galton, began developing mathematical tools to analyse statistical data. The most famous of these tools, namely, the X^2 -test of goodness of fit, has survived the passage of time and has been hailed as one of the major innovations of the last hundred years. It is in Pearson's early work, more than anywhere else, that one finds the beginning of the modern discipline of statistics. Most of Pearson's own work, and the other work that it inspired, appeared in the journal *Biometrika*, which was founded by Pearson in 1901. It was through this journal in 1915 that Prasanta Chandra Mahalanobis had his first initiation in Statistics.

Mahalanobis, born in a well-to-do progressive Brahmo family in 1893, had gone to study in Cambridge in 1913 and in 1915 had just finished his Tripos in physics. This is how Professor C.R. Rao describes his first encounter with *Biometrika*¹:

'At the time of Mahalanobis's departure to India from Cambridge, the first world war was on and there was a short delay in his journey. Mahalanobis utilized this time browsing in the King's College library. One morning Macaulay, the tutor, ... drew his attention to some bound volumes of *Biometrika*.... Mahalanobis got so interested that he bought a complete set of *Biometrika* volumes... he started reading the volumes on the boat during his journey and continued to study and work out exercises on his own during spare time after arrival in Calcutta. He tried to look for problems where he could apply the new knowledge he was acquiring'.

In these pursuits, Mahalanobis was encouraged by Acharya Brajendra Nath Seal, who had been one of the first men to appreciate the significance of the new discipline.

In 1917, Seal, as Chairman of a Committee for examination reforms in Calcutta University, sought Mahalanobis's help in statistical analysis of examination results. This was probably the first important work in Statistics in the modern sense to be undertaken in India. But unfortunately Seal left Calcutta to take up a new job as Vice-Chancellor at Mysore University, and his committee did not finish its task or submit a report. I have been able to trace a letter of 23 May 1971 from Seal in which Seal describes in great statistical detail exactly what he wants to be done but except for this no record has survived². From 1919 onwards we have records of Mahalanobis's work, though occasionally only in the form of an abstract in the proceedings of the Indian Science Congress.

Mahalanobis set up the Statistical Laboratory in the Presidency College sometime in the twenties*. In 1931, the Indian Statistical Institute was founded as a Registered Society and appears to have been housed at the Statistical Laboratory. All or nearly all the statistical work that was being done in India in the twenties and thirties was done by Mahalanobis and his students or colleagues at the Statistical Laboratory. In the thirties, other distinguished work, notably by Raj Chandra Bose, Samarendra Nath Roy, appeared on the scene for the first time, but Mahalanobis continued to be the leading figure either through his own work or by virtue of the influence he exerted on the choice of problem areas. Both Bose and Roy worked on problems of determining the central and non-central sampling distribution of Mahalanobis's studentised and non-studentised D^2 -statistics and Bose's pathbreaking work on design of experiments in the late thirties would not have been possible but for Mahalanobis's interest and pioneering work in introducing the new designs of Fisher to Indian agricultural field trials.

The situation began to change in the forties, but much remained the same. Important work had begun to appear elsewhere most notably at the Statistical Section of the ICAR (then the Imperial Council of Agricultural Research) which was to evolve, through various stages, into a separate research institute in the late fifties. However, it was still the work in Calcutta, at the Indian Statistical Institute, that mattered most. The Indian Statistical Institute was now internationally known and was taken as a model when the first institute in Statistics was set up in the United States by Gertrude Cox – perhaps the only time an institute in a developing country was used as a model in a developed country. Though new directions of research had emerged in the Institute, Mahalanobis's own work, for example, his long classic paper on large scale sample surveys, which won him an FRS, was as important as the best work of Bose, Roy and C. Radhakrishna Rao, a new brilliant young man at the Institute, who was himself to be one of the most famous statisticians of our time.

In many ways the end of the forties was to be a watershed. First and foremost, there were new independent centres of research in and outside Calcutta. For the

* I have not been able to ascertain the exact dates but the Laboratory was in existence from the late twenties to probably early fifties when the Indian Statistical Institute moved to its present campus at Baranagar.

Institute itself also, it was a time of great change, both intellectually and physically. Bose and Roy left India and settled down in the United States. Rao provided leadership to the newly set up Research and Training School of the Institute, which, according to the new constitution, was to house all the major training and theoretical and methodological research activities. The government, which was increasingly bearing a large share of the Institute's expenses, had wanted it to concentrate only on training and research in statistics. But Mahalanobis fought against what he thought was a wrong perception of the role of the Institute as well as the discipline. After lengthy negotiations with the government which went on for nearly a decade and in which the Institute's President and friend C.D. Deshmukh had to intercede several times on behalf of the Institute, Mahalanobis was finally able to persuade the government that the Institute must also be allowed to take up large scale useful or pioneering projects, as it had always done, for divorced from practical work, research and training might become too bookish. He also won his case for autonomy of the Institute in its day-to-day administration. Such battles with the government would plague him and the Institute ever after – perhaps in a developing country there will always be an uneasy tension between a powerful but uninformed government and an institute which tries to be different in a world of mediocrity or worse.

Academically also much was to change in the fifties. Mahalanobis would remain creative, producing some of his best and most well-known work, for example, the two-sector planning model for India, but it would hardly be called Statistics in the sense in which the discipline was understood internationally among academics and it had little or no relation to the important new theoretical or methodological research going on under the leadership of Rao. Though the fifties would be regarded by many as the golden period of the Institute, the growth of the subject and the growth of the Institute were no longer as closely integrated as before and this element of contradiction would introduce new complexities and tensions.

Statistics continued to grow very fast and frequently got compartmentalised in emerging, self-contained, almost non-interacting areas, like probability and inference, including sampling and multivariate analysis, construction of designs, econometrics and demography, the last two being almost new disciplines in their own right. Growth of the subject as well as growth of the Institute created new opportunities as well as structural problems. The separate, often unrelated, development of theoretical and practical work was a new problem that would remain with the Institute, though Rao himself, the architect of the Research and Training School, was as comfortable in one as in the other. Indeed the gap between theory and practice was a problem with the subject itself and only the late seventies and eighties saw the beginning of an attempt to recover the lost unity. But that is a different story.

In the next two sections, we return to the period under review for a more detailed look at some of the things that were happening then.

2: MAHALANOBIS AND THE TWENTIES

2.1 Nothing makes more clear the extent to which Mahalanobis dominated organisation

of research in Statistics than an examination of the important events in the history of statistical work in India. Here are some important events and dates, as compiled in Dr Rao's *Ramanujan Lectures*³.

- 1895 Establishment of the Statistical Bureau in Calcutta.
- 1905 Establishment of the Department of Commercial Intelligence and Statistics in Calcutta.
- 1931 Founding of the Indian Statistical Institute (ISI)
- 1931 Creation of the Imperial (now) Indian Council of Agricultural Research (ICAR) with a statistics branch.
- 1933 Starting of *Sankhyā*, the Indian Journal of Statistics.
- 1938 First All Indian Statistical Conference.
- 1939 One year training course in statistics started by the ISI.
- 1941 Calcutta University introduces graduate courses, for the first time in any Indian university, leading to Master's Degree in Statistics.
- 1942 Indian Science Congress includes, for the first time, Statistics as a part of the section on Mathematics.
- 1945 Mahalanobis is elected a Fellow of the Royal Society, UK for his contributions to Statistics.
- 1950 Establishment of the National Sample Survey, the first large scale multipurpose survey in the world.
- 1951 27th session of the International Statistical Institute held in New Delhi
- 1951 Establishment of the Central Statistical Organization and State Statistical Bureaus.

To understand the full significance of these events in the present context, it must be remembered that it was Mahalanobis who organised the first national conference in Statistics. He was the first statistician to preside over the section of Mathematics and Statistics at the annual session of the Indian Science Congress in 1942, and was instrumental in the addition of a separate section on Statistics from 1945 onwards to the annual sessions of the Indian Science Congress. National Sample Survey was his brainchild and he was probably the guiding spirit, as well as one of the chief architects, of the Central Statistical Organisation.

A record of these activities appears in Mahalanobis's own reminiscences⁴ as well as the biographical sketches by his cousin Anikendra Mahalanobis⁵ and Professor C.R. Rao⁶. Considering the fact that it was written by a non-statistician, the first is a remarkably complete account of the main events in Mahalanobis's life as well as his major achievements as a scientist and science administrator. The second is a marvellous but honest tribute of one creative scientist to another. One may also consult the history of the activities of the Indian Statistical Institute, its annual reports during the years 1931 to 1950⁷, and Mahalanobis's private papers, now in the custody of the Nehru Memorial Museum and Library and the Indian Statistical Institute.

A man of renaissance proportions emerges from these records. Even in the twenties, when he is establishing himself in a new discipline, Statistics is only one of his many

passions. He is in love with Nirmal Kumari, his future wife and daughter of the redoubtable Heramba Chandra Maitra, with whom he is waging an ideological battle for the reform of the Brahma Samaj. When Heramba Chandra finally relents and lets Mahalanobis marry his daughter, they have only his permission, not his approval. The bride is given away by Mahalanobis's maternal uncle, Sir Nilratan Sircar, in a ceremony attended by Rabindranath. The poet gives to the couple the manuscript of his new dance drama *Basanta* as a wedding gift. He is at the poet's side in 1919 when Rabindranath wrote his historic protest letter against the Jalianwalabag massacre to Lord Chelmsford, renouncing his Knighthood. Throughout the twenties, he is one of the Secretaries of the Visva-Bharati as well as a close and trusted companion of Rabindranath. He is also one of Sukumar Ray's best friends and it is to him that Ray confides, in one of his last letters, about his crisis in faith.

A charming penportrait of him survives in a letter of Brojendranath Seal written to Mahalanobis from Gujarat College, Ahmedabad, in 1921.

'He tells me you intend to live in the wilds and fill your brains with the statistics of rainfall, temperature and other mysterious attributes. The job will suit you and your temperament. I always had an ethereal impression of you: tall, thin and lanky, elusive and shall I say enigmatic!

This tall, lanky man, with catholic tastes and varied interests, had, of course, more than a man's normal share of toughness, and he knew it. Years later, in response to a young colleague's query as to what was the most important quality for a great administrator to have, he would reply, 'The capacity to be unpleasant when the occasion called for it'⁸. He was also very tough as an intellectual with a clear vision and faith in his vision. During the negotiations with the government in the forties, he offered to resign and start a new institute rather than accept directorship of an organisation which would have to deviate from his vision under pressure from the government⁹.

Though Statistics was only one of his many loves in the twenties, it was also his most abiding and most important. In one of the rare singularities of history, the subject, the man and his times fitted one another absolutely well. Professor Ashok Rudra, in the course of delivering a Mahalanobis Memorial Lecture, wondered as to the reasons behind Mahalanobis's uncanny success as a science administrator and whether by studying his life one could repeat his success. While singularities in history are not easy to repeat, one can certainly learn a lot from Mahalanobis's experiences and what Mahalanobis himself wrote about these matters¹⁰. Much of what Mahalanobis wrote about prerequisites for progress of science in developing countries remains quite relevant, indicating how little we have progressed towards the resolution of basic issues. Before I pass on to review Mahalanobis's scientific work in the twenties, let me quote once more from Professor Rao¹¹.

'Such was the man who combined his great intellect and vision with an unlimited capacity for work and brought reputation to the country

by his achievements. No one in history could achieve anything great unless he was tough, could act boldly with faith in his convictions, and had the ability to argue ... and get things done. Mahalanobis had all these traits in good measure...

...‘Statistical science was a virgin field and practically unknown in India before the twenties.... It needed a pioneer and adventurer like him, with courage and tenacity to fight all opposition ...’

‘In India, traditionally there have been no organization set-ups to cause progress in knowledge and advance society. In countries where such organizations exist, individuals have only been instrumental for action. The reverse has been true in India. Individuals dominated the scene, as even now, both in science and in politics...’

‘Mahalanobis was no exception. He was the boss of the Institute, and an ‘impossible boss’ according to some... but we do *not* have any example in India where collective thinking and multilevel responsibilities have yielded fruitful results in public enterprises or educational institutions’.

2.2 Let me now examine briefly some of his pathbreaking papers in Statistics in the twenties.

As a scientist, Mahalanobis combined an exceptional flair for empirical studies, especially those which are necessary and important for understanding a society and planning and making decisions for it. He brought to his studies an unerring intuition for capturing the essence of a problem and a genius for evolving simple but effective tools for analysing it. All these early studies exhibit his confidence in his own analysis and optimism about beneficial applications to society. This was not, however, the confidence and optimism of the unwary. For, his techniques on the whole have stood the test of time well, even where subsequent accumulation of data has led to modification of some of his conclusions.

In the paper entitled “Analysis of Race-mixture in Bengal” published in 1925¹², Mahalanobis was trying to answer questions of the following kind. How are the anglo-Indians of Calcutta related to different caste groups of Bengal? Are they more closely allied with the Hindus? Do they show a greater affinity with the higher castes of Bengal or with the lower castes? In order to answer these questions he then proceeds to study what becomes the central theme of the paper, namely, ‘the geographical and social resemblances shown by the typical Bengal castes whose antecedents and present status are fairly well-known’.

In order to carry out this study, he uses, probably for the first time, the Mahalanobis D^2 , recording his reasons for preferring it to a similar coefficient of Karl Pearson. The formula for D^2 does not yet take into account the correlation between the variables S

measured – that final evaluation takes place in a paper published in the thirties¹³. Nor has he yet worked out the sampling distribution of the non-studentised D^2 . This is guessed in¹⁴ through calculation of moments and matching with Pearsonian curves – the guess is right for the central case and a reasonable approximation in the non-central case. Sampling distributions of the studentised D^2 in the non-central case, which is one of the famous results of the Indian school, was worked out towards the end of the thirties by Bose and Roy. Throughout thirties and forties there were important contributions to the use of the D^2 . Even to-day it remains one of the major tools in problems of classification and cluster analysis, and continues to be applied in anthropology and other areas. There is evidence (Pearson's letter to Mahalanobis in the ISI collection and Mahalanobis's letter to Fisher in the Nehru Memorial Museum and Library Collection) that Mahalanobis had received a rebuff from Pearson who had expressed reservations about the D^2 statistics. Yet, as subsequent events have shown, if Mahalanobis had done nothing else but this, he would still have been a famous statistician.

How well have the conclusions of the 1925 paper survived the test of time? The important frequently quoted conclusion that the Bengal Brahmans resemble other Bengal castes far more closely than they resemble Brahmans elsewhere in India, is still quite valid. However, the other significant conclusion, namely, that the Brahmans alone can justifiably claim definite connections with upper India, particularly with the Punjab, has not been borne out by subsequent data. Moreover, as far as the Anglo-Indian community is concerned, it is now believed that Mahalanobis had probably confined his study to a sample from the upper stratum of the community and hence his conclusion of resemblance to upper caste Hindus is applicable to the upper class Anglo-Indians only.

In addition to his anthropometric studies, Mahalanobis initiated statistical applications to meteorology and flood control. In 1922, after a disastrous flood in North Bengal, Mahalanobis was asked to examine a recommendation to construct expensive retarding basins to hold up the flood water. After a detailed study of the 50-year data on rainfall and flood, he proposed entirely different measures which were implemented and proved effective¹⁵. A little later, after a severe flood in the Brahmini river, the question of flood control was again referred to Mahalanobis. Earlier, an expert committee of engineers had thought that a catastrophic rise had occurred in the river bed, which could be offset by increasing the height of the embankments. Again, study of the 60-year data did not show any change that had taken place in the river bed and a better idea was to construct dams in the upper reaches of the river. In the course of this study, Mahalanobis made calculations which formed the basis of the Hirakud hydroelectric project. Similar studies in the thirties were used in the Damodar valley projects¹⁶.

Most of this work appeared in the long official reports submitted to the government, but an interested reader can find some of the statistical findings and techniques used in these reports in a paper published in 1940¹⁷.

These studies on flood control were probably pioneering studies in operations research. They helped to prepare Mahalanobis for his later work on large scale sample surveys and planning, where the ideas of operations research appear quite explicitly.

3: STATISTICS COMES OF AGE – THIRTIES AND FORTIES.

At a public meeting on 17 December 1931, with the late Sir R.N. Mukherjee in the chair, it was decided to establish the Indian Statistical Institute as a learned society. The Institute was registered on 28 April 1932. *Sankhyā, the Indian Journal of Statistics*, started appearing from 1933.

The outstanding young men who either joined the Institute on their own or were recruited by Mahalanobis, included S.S. Bose, who had been with him since the twenties, J.M. Sengupta whose flair for survey work was to become legendary¹⁸, K.R. Nair, R.C. Bose and S.N. Roy. Bose was recruited as a promising young geometer in 1933 and Roy came within a year. Together they founded the Indian School of Theoretical Research on Multivariate Analysis – an area where Indian contribution continues to be significant. In the thirties, their work culminated in the discovery of the sampling distribution of the studentised D^2 as a non-central F-distribution¹⁹. This was a deep result obtained by hard, innovative mathematics, but had its origin in the questions raised by Mahalanobis.

Fisher, who first visited the Institute around this time to attend the first national conference in India on Statistics, was very excited by this result of Bose and Roy. His reactions about this as well as the very favourable impressions he formed about the Institute have been recorded by his daughter in her biography of Fisher²⁰. Fisher's visit was a turning point in the career of Bose who got interested in the combinatorial problems of experimental design from this time and almost immediately published an outstanding paper on this topic in the *Annals of Eugenics*, which was edited by Fisher²¹. Throughout forties and later, this was to be the area that Bose made his own. Fisher also had a great influence on Rao who joined the Institute in 1941 and later went to Cambridge to study with Fisher for his doctorate. Through both Bose and Rao, Fisher exerted a strong influence on the Indian School. It is, however, remarkable that in spite of their close friendship, Mahalanobis's work continued to develop along independent lines. Of course, he must have been influenced by Fisher's philosophy, but if he can be said to have learnt statistics from anyone, it must be Karl Pearson whom Mahalanobis continued to respect in spite of their differences over the use of D^2 . On hearing of Pearson's death, he wrote to his son,

'I came in touch with him only for a few months but I have always looked upon him as my master and, myself as one of his humble disciples'²².

Apart from intellectual enrichment, Fisher's visit helped the Institute in two ways. It brought international recognition and enhanced its prestige in the eyes of the Indian

Government. This must have been useful during the difficult negotiations with the government throughout the forties.

Two other men helped the Institute a lot, Jawaharlal Nehru who first visited the Institute in the forties and Chintaman Dwarknath Deshmukh who became the Institute's President during a critical period in the forties.

In the thirties, Mahalanobis's interest had begun to shift from multivariate analysis to large scale sample surveys. Possibly, the most important of these was an exploratory sample survey of the jute crop in 1937, which became a five year project and was completed in 1941-42. This and other surveys provided the basis for the fundamental paper on methodology as well as philosophical issues involved in Survey Sampling²³. In the matter of sample survey techniques, his most well-known contributions are the concept of optimum survey design, interpenetrating samples, and pilot surveys. Abraham Wald, who developed statistical decision theory and technique of sequential analysis and was the most well-known theoretical statistician in the forties, recognized in pilot surveys the seminal ideas of sequential analysis. In addition to introducing these techniques, Mahalanobis raised important difficult philosophical issues of randomness and representativeness of a sample, which remain relevant and challenging to contemporary theoreticians. Mahalanobis contributions to large scale sample surveys, whether through this paper or the National Sample Survey which he set up in 1950 or the U.N. Sub-Commission in Sampling to which he was a member from 1947 to 1951, are among his most significant and lasting gifts to our subject.

A great deal of very important theoretical work was done by others also in the forties – much of it inspired by Bose and Roy. Roy continued to work on multivariate analysis, but Bose had shifted to construction of designs. In this new area, with Nair and Kishen or alone, Bose wrote several classic papers and set up a tradition of research which remains vigorous to this day. Sampling, design of experiments and multivariate analysis are still the favourite topics of research in India. Rao, with a path-breaking paper on minimum attainable variance for unbiased estimates, was opening up entirely new directions of research in inference and design. Among others in the Institute one must mention A. Bhattacharya, the demographer, Ajit Dasgupta and the number theorist turned sampling expert, D.B. Lahiri. The first Department of Statistics was opened in Calcutta University in 1941, with Mahalanobis as the honorary head. Towards the end of the decade, the department was functioning independently of the Institute and had on its faculty two brilliant young men, M.N. Ghosh and H.K. Nandi. Ghosh had extended Wald's decision theory and Nandi had contributed to sequential analysis and testing as well as design of experiments. It was clear that the best young minds of the time were being attracted to the emerging problems of statistical inference and probability.

At the end of the decade, research in Statistics was no longer the work of a single man or a single Institute. Statistical research had come of age.

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