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Research Paper

Upper Limits to the Complex Growth Rates in Triply Diffusive Convection

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The paper mathematically establishes that the complex growth rate (p_r, p_i) of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude, in a triply diffusive fluid layer with one of the components as heat with diffusivity κ, must lie inside a semicircle in the right-half of the (p_r, p_i)-plane whose centre is origin and radius equals $\sqrt{(R_1 + R_2)\sigma}$ where, R_1 and R_2 are the Rayleigh numbers for the two concentration components with diffusivities κ_1 and κ_2 (with no loss of generality, $\kappa > \kappa_1 > \kappa_2$) and σ is the Prandtl number. Further, it is proved that above result is uniformly valid for quite general nature of the bounding surfaces.

Key Words: Triply Diffusive Convection; Oscillatory Motions; Complex Growth Rate

1. Introduction

Convective motions can occur in a stably stratified fluid when there are two components contributing to the density which diffuse at different rates. This phenomenon is called double-diffusive convection. To determine the conditions under which these convective motions will occur, the linear stability of two superposed concentration (or one of them may be temperature gradient) gradients has been studied by Stern (1960), Veronis (1965), Nield (1967), Baines and Gill (1969) and Turner (1968) etc.

All these researchers have considered the case of two component systems. However, it has been recognized later on (Griffiths 1979, Turner 1985) that there are many situations wherein more than two components are present. Examples of such multiple diffusive convection fluid systems include the solidification of molten alloys, geothermally heated lakes, magmas and their laboratory models and sea water. Griffiths (1979), Pearlstein *et al*. (1989) and Lopez (1990) have theoretically studied the onset of convection in a horizontal layer, of infinite extension, of a triply diffusive fluid (where the density depends on three independently diffusing agencies with different diffusivities). These researchers found that small concentrations of a third component with a smaller diffusivity can have a significant effect upon the nature of diffusive instabilities and ‘oscillatory’ and direct ‘salt finger’ modes are simultaneously unstable under a wide range of conditions, when the density gradients due to components with the greatest and smallest diffusivity are of same signs. Some fundamental differences between the double and triply diffusive convection are noticed by these researchers. Among these differences one is that if the gradients of two of the stratifying agencies are held fixed, then three critical values of the Rayleigh number of the third agency are sometimes required to specify the linear stability criteria (in double diffusive convection only one critical Rayleigh number is required). Another is that the onset of convection may occur via a quassiperiodic bifurcation from the motionless basic state. Ryzhkov and Shevtsova (2007) studied the case of multicomponent mixture with application to the thermogravitational

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column. Ryzhkov and Shevtsova (2009) also studied the longwave instability of a multicomponent fluid layer with the Soret effect. Salvatore Rionero (2013a) studied a triple convective diffusive fluid mixture saturating a porous horizontal layer, heated from below and salted from above and obtained sufficient conditions for inhibiting the onset of convection and guaranteeing the global nonlinear stability of the thermal conduction solution. Salvatore Rionero (2013b) also investigated the multicomponent diffusive convection in porous layer for the more general case when heated from below and salted by m salts partly from above and partly from below.

The problem of obtaining bounds for the complex growth rate of an arbitrary oscillatory perturbation of growing amplitude in triply diffusive convection problem is an important feature of fluid dynamics, especially when both the boundaries are not dynamically free so that exact solutions in the closed form are not obtainable, the bounds for the complex growth rate of an arbitrary oscillatory perturbation of growing amplitude in triply diffusive case must be found. We prove the following theorem, as a first step, in this direction:

The complex growth rate (p_r, p_i) of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude, in a triply diffusive fluid layer with one of the components as heat with diffusivity κ, must lie inside a semicircle in the right-half of the (p_r, p_i)-plane whose centre is origin and radius equals $\sqrt{(R_1+R_2)\sigma}$ where, R_1 and R_2 are the Rayleigh numbers for the two concentration components with diffusivities κ_1 and κ_2 (with no loss of generality, $\kappa > \kappa_1 > \kappa_2$) and σ is the Prandtl number. Further, it is proved that above result is uniformly valid for quite general nature of the bounding surfaces and the results of Banerjee *et al*. (1981) for double diffusive convection follow as a consequence.

2. Mathematical Formulation and Analysis

A viscous finitely heat conducting Boussinesq fluid of infinite horizontal extension is statistically confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and T_1 ($< T_0$) and uniform concentrations S_{10}, S_{20} and S_{11} ($<S_{10}$), S_{21} ($< S_{20}$) (as shown in Fig. 1).

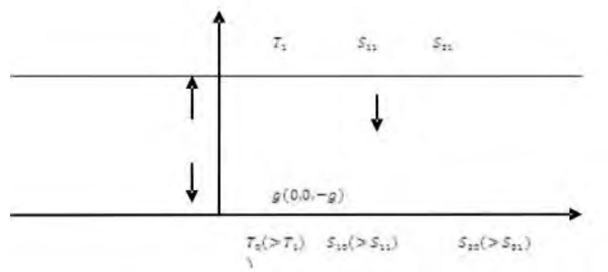

Fig. 1: Physical configuration

The basic equations that govern the motion of triply diffusive fluid layer are as follows:

The equation of continuity is

$$\frac{\partial\rho}{\partial t}+u_j\frac{\partial\rho}{\partial x_j}+\rho\frac{\partial u_j}{\partial x_j}=0 \tag{1}$$

The equation of motion is

$$\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=\rho X_i-\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_i}\left\{\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{2}{3}\mu\delta_{ij}\frac{\partial u_k}{\partial x_k}\right\} \tag{2}$$

The equation of heat conduction is

$$\frac{\partial(\rho C_v T)}{\partial t}+\frac{\partial(\rho C_v T u_j)}{\partial x_j}=\frac{\partial}{\partial x_j}\left(K\frac{\partial T}{\partial x_j}\right)-P\frac{\partial u_j}{\partial x_j}+\Phi, \tag{3}$$

where, $\Phi = 2\mu e_{ij}^2-\frac{2}{3}e_{ij}^2$ and $e_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$

Equation (3), with the help of (1), can be simplified to the form

$$\rho\left[\frac{\partial(C_v T)}{\partial t}+u_j\frac{\partial(C_v T)}{\partial x_j}\right]=\frac{\partial}{\partial x_j}\left(K\frac{\partial T}{\partial x_j}\right)-P\frac{\partial u_j}{\partial x_j}+\Phi. \tag{4}$$

The equations of mass diffusion are

$$\frac{\partial S_1}{\partial t}+u_j\frac{\partial S_1}{\partial x_j}=\kappa_1\nabla^2 S_1, \tag{5}$$

and

$$\frac{\partial S_2}{\partial t}+u_j\frac{\partial S_2}{\partial x_j}=\kappa_2\nabla^2 S_2, \tag{6}$$

where, ρ is density, t is time, x_j (j = 1, 2, 3) are Cartesian coordinates x, y, z; u_j (j = 1, 2, 3) are velocity components; X_i (i = 1, 2, 3) are the components of external force in x, y, z directions respectively; P is the pressure, μ is viscosity, C_v is the specific heat at constant volume, T is the temperature, K is heat conductivity; S_1, S_2 are the two concentrations and κ_1, κ_2 are respectively the coefficients of mass diffusivity of S_1, S_2 with $\kappa_1 > \kappa_2$.

The above basic equations must be supplemented by equation of state

$$\rho=\rho_0[1+\alpha(T_0-T)-\alpha'(S_{10}-S_1)-\alpha''(S_{20}-S_2)], \tag{7}$$

where, α, α' and α'' are respectively the coefficients of volume expansion due to temperature variation and concentration variation for the two concentration components S_1 and S_2. ρ_0 is the value of ρ at z = 0.

Applying the usual Boussinesq approximations which in essence amounts to neglecting terms which are of order 10^{-3} at the most as compared to 1 for variations in temperature of only moderate amounts, we obtain the simplified forms of equations (1), (2), (4), (5) and (6) as

$$\frac{\partial u_i}{\partial x_j}=0, \tag{8}$$

$$\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{1}{\rho_0}\frac{\partial P}{\partial x_i}+\left(1+\frac{\delta\rho}{\rho_0}+\frac{\delta\rho'}{\rho_0}+\frac{\delta\rho''}{\rho_0}\right)X_i+\nu_0\nabla^2 u_i, \tag{9}$$

$$\frac{\partial T}{\partial t}+u_j\frac{\partial T}{\partial x_j}=\kappa_0\nabla^2 T, \tag{10}$$

$$\frac{\partial S_1}{\partial t}+u_j\frac{\partial S_1}{\partial x_j}=\kappa_{10}\nabla^2 S_1, \tag{11}$$

and

$$\frac{\partial S_2}{\partial t}+u_j\frac{\partial S_2}{\partial x_j}=\kappa_{20}\nabla^2 S_2, \tag{12}$$

where, $\nu_0=\dfrac{\mu}{\rho_0}$ is kinematic viscosity, $\kappa_0=\dfrac{K}{\rho_0 C_{v0}}$ is thermal diffusivity.

Now the initial state solutions on the basis of initial state $(u, v, w) \equiv (0, 0, 0)$, $T \equiv T(z)$, $S_1 \equiv S_1(z)$, $S_2 \equiv S_2(z)$ and $\rho = \rho(z)$ is given by

$$(u,v,w)=(0,0,0), T=T_0-\beta z, S_1=S_{10}-\beta' z,$$
$$S_2=S_{20}-\beta'' z, \rho=\rho_0[1+\alpha\beta z-\alpha'\beta' z-\alpha''\beta'' z]$$

and

$$P=P_0-g\rho_0\left[z+(\alpha\beta-\alpha'\beta'-\alpha''\beta'')\frac{z^2}{2}\right], \tag{13}$$

where, $\beta=\dfrac{T_0-T_1}{d}$ is the maintained uniform adverse temperature gradient, $\beta'=\dfrac{S_{10}-S_{11}}{d}$ and $\beta''=\dfrac{S_{20}-S_{21}}{d}$ are the maintained non-adverse concentration gradients and P_0 is the value of P at z = 0.

To study the stability of the system, we perturb all the variables in the form

$$
\begin{aligned}
&(\bar{u},\bar{v},\bar{w}) = (0+u',0+v',0+w'),\\
&\bar{T} = T_0 - \beta z + \theta', \bar{S}_1 = S_{10} - \beta' z + \phi_1'\\
&\bar{S}_2 = S_{20} - \beta'' z + \phi_2', \bar{\rho} = \rho_0\\
&[1+\alpha(T_0 - T - \theta') - \alpha'(S_{10} - S_1 - \phi_1')\\
&-\alpha''(S_{20} - S_2\phi_2')] \text{ and } \bar{P} = P_0 - g\rho_0 \qquad (14)\\
&\left[z + (\alpha\beta - \alpha'\beta' - \alpha''\beta'')\frac{z^2}{2}\right] + \delta P',
\end{aligned}
$$

where, $u', v', w', \phi_1', \phi_2'$ and $\delta P'$ are perturbed variables and are assumed to be small. Substituting (14) into Eqs. (8)-(12), we obtain the following linearized perturbation equations

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0, \qquad (15)$$

$$\rho_0 \frac{\partial u'}{\partial t} = -\frac{\partial \delta p'}{\partial x} + \mu_0 \nabla^2 u', \qquad (16)$$

$$\rho_0 \frac{\partial v'}{\partial t} = -\frac{\partial \delta p'}{\partial y} + \mu_0 \nabla^2 v', \qquad (17)$$

$$
\begin{aligned}
\rho_0 \frac{\partial w'}{\partial t} = -\frac{\partial \delta p'}{\partial z} + \mu_0 \nabla^2 w', + g\rho_0\alpha\theta'\\
- g\rho_0\alpha'\phi_1' - g\rho_0\alpha''\phi_2',
\end{aligned} \qquad (18)
$$

$$\frac{\partial \theta'}{\partial t} - \beta w' = \kappa_0 \nabla^2 \theta', \qquad (19)$$

$$\frac{\partial \phi_1'}{\partial t} - \beta' w' = \kappa_{10} \nabla^2 \phi_1', \qquad (20)$$

and

$$\frac{\partial \phi_2'}{\partial t} - \beta'' w' = \kappa_{20} \nabla^2 \phi_2', \qquad (21)$$

The normal mode expansion of the dependent variables u', v', w', θ', ϕ_1', ϕ_2' and $\delta P'$ is assumed in the form

$$F'(x,y,z,t) = F''(z)\exp[i(k_x x + k_y y) + nt], \qquad (22)$$

where, $k = \sqrt{k_x^2 + k_y^2}$ is the wave number of perturbation, k_x, k_y, being real constants and n is a constant which can be complex in general.

For functions with this dependence on *x*, *y* and *t* we have

$$\frac{\partial}{\partial t} = n, \ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = -k^2 \text{ and } \nabla^2 = \frac{d^2}{dz^2} - k^2. \qquad (23)$$

Eqs. (15)- (21), then become

$$ik_x u'' + ik_y v'' + \frac{dw''}{dz} = 0 \qquad (24)$$

$$\rho_0 n u'' = -ik_x \delta P'' + \mu_0 \left(\frac{d^2}{dz^2} - k^2\right) u'', \qquad (25)$$

$$\rho_0 n v'' = -ik_x \delta P'' + \mu_0 \left(\frac{d^2}{dz^2} - k^2\right) v'', \qquad (26)$$

$$
\begin{aligned}
\rho_0 n w'' = -\frac{d}{dz}(\delta P') + \mu_0 \left(\frac{d^2}{dz^2} - k^2\right) w''\\
+ g\rho_0\alpha\theta'' - g\rho_0\alpha'\phi_1'' - g\rho_0\alpha''\phi_2''
\end{aligned} \qquad (27)
$$

$$n\theta'' - \beta w'' = \kappa_0 \left(\frac{d^2}{dz^2} - k^2\right)\theta'', \qquad (28)$$

$$n\phi_1'' - \beta' w'' = \kappa_{10} \left(\frac{d^2}{dz^2} - k^2\right)\phi_1'', \qquad (29)$$

$$n\phi_2'' - \beta'' w'' = \kappa_{20} \left(\frac{d^2}{dz^2} - k^2\right)\phi_2'', \qquad (30)$$

Eliminating u'' and v'' between Eqs. (25) and (26) by using Eq. (24) and then eliminating between this resulting equation and Eq. (27), we obtain the equation

$$\left(\frac{d^2}{dz^2}-k^2\right)\left(\frac{d^2}{dz^2}-k^2-\frac{n}{\nu_0}\right)w''$$
$$=\frac{g\alpha k^2\theta''}{\nu_0}-\frac{g\alpha' k^2\phi_1''}{\nu_0}-\frac{g\alpha'' k^2\phi_2''}{\nu_0} \quad (31)$$

Eqs. (28)-(30) can be written as

$$\left(\frac{d^2}{dz^2}-k^2-\frac{n}{\kappa_0}\right)\theta''=\frac{\beta}{\kappa_0}w'', \quad (32)$$

$$\left(\frac{d^2}{dz^2}-k^2-\frac{n}{\kappa_{10}}\right)\phi_1''=\frac{\beta'}{\kappa_{10}}w'', \quad (33)$$

$$\left(\frac{d^2}{dz^2}-k^2-\frac{n}{\kappa_{20}}\right)\phi_2''=\frac{\beta''}{\kappa_{20}}w'' \quad (34)$$

Now by introducing the non-dimensional quantities defined by

$$z_*=\frac{z}{d},\ \tau_{1*}=\frac{\kappa_{10}}{\kappa_0},\ \tau_{2*}=\frac{\kappa_{20}}{\kappa_0},$$

$$\sigma_*=\frac{\nu_0}{\kappa_0},\ D_*=d\frac{d}{dz},\ p_*=\frac{nd^2}{\kappa_0},\ a_*=kd$$

and

$$R_*=\frac{g\alpha\beta d^4}{\kappa_0\nu_0}, R_{1*}=\frac{g\alpha'\beta' d^4}{\kappa_0\nu_0}, R_{2*}=\frac{g\alpha''\beta'' d^4}{\kappa_0\nu_0},$$

$$w_*=\frac{d}{\kappa_0}w'', \theta_*=\frac{\theta''}{\beta d}, \phi_{1*}=\frac{\phi_1''}{\beta' d}, \phi_2=\frac{\phi_2''}{\beta'' d} \quad (35)$$

we can reduce Eqs. (31)-(34) into the following non-dimensional forms (dropping the asterisks for convenience):

$$(D^2-a^2)\left(D^2-a^2-\frac{p}{\sigma}\right)w$$
$$=Ra^2\theta-R_1a^2\phi_1-R_2a^2\phi_2 \quad (36)$$

$$(D^2-a^2-p)\theta=-w, \quad (37)$$

$$\left(D^2-a^2-\frac{p}{\tau_1}\right)\phi_1=-\frac{w}{\tau_1}, \quad (38)$$

$$\left(D^2-a^2-\frac{p}{\tau_2}\right)\phi_2=-\frac{w}{\tau_2}, \quad (39)$$

Eqs. (36)-(39) are to be solved using the following appropriate boundary conditions:

$w=0=\theta=\phi_1=\phi_2=Dw$ at $z=0$

and $z=1$, (both the boundaries are rigid) (40)

or $w=0=\theta=\phi_1=\phi_2=D^2w$ at $z=0$

and $z=1$, (both the boundaries are free) (41)

or $w=0=\theta=\phi_1=\phi_2=Dw$ at

(lower boundary is rigid)

and $w=0=\theta=\phi_1=\phi_2=D^2w$

at $z=1$ (upper boundary is free) (42)

or $w=0=\theta=\phi_1=\phi_2=D^2w$ at $z=0$

(lower boundary is free)

and $w=0=\theta=\phi_1=\phi_2=Dw$ at $z=1$

(upper boundary is rigid) (43)

The meaning of the symbols involved in Eqs. (36)-(43) from the physical point of view are as follows: z is the vertical coordinate, D is the differentiation w.r.t. z, a^2, is square of the wave number, $\sigma>0$ is the Prandtl number, $\tau_1>0$ and $\tau_2>0$ are the Lewis numbers for the two concentrations S_1 and S_2 respectively, R is the thermal Rayleigh number, R_1 and R_2 are concentration Rayleigh numbers for the two concentration components, $p=p_r+ip_i$ is the complex growth rate where p_r and p_i are real constants, w is the vertical velocity, θ is the temperature ϕ_1 and ϕ_2 are the respective concentrations of the two components. It may further be noted that Eqs. (36)-(43) describe an eigen value problem for and govern triply diffusive convection for any combination of dynamically free and rigid boundaries.

We prove the following theorem:

Theorem 1: If $R > 0$, $R_1 > 0$, $R_2 > 0$, $p_r \geq 0$ and $p_r \neq 0$ then a necessary condition for the existence of a nontrivial solution (w, θ, ϕ_1, ϕ_2, p) of Eqs. (36)-(39) together with boundary conditions (40) or (41) or (42) or (43) is that

$$|p|^2 < (R_1 + R_2)\sigma. \tag{44}$$

Proof : Multiplying Eq. (36) by w^* (the complex conjugate of w) throughout and integrating the resulting equation over the vertical range of z, we have

$$\int_0^1 w^*(D^2-a^2)\left(D^2-a^2-\frac{p}{\sigma}\right)wdz =$$

$$Ra^2\int_0^1 w^*\theta\, dz - R_1a^2\int_0^1 w^*\phi_1 dz - R_2a^2\int_0^1 w^*\phi_2 dz. \tag{45}$$

Using Eqs. (37), (38) and (39), we can write

$$Ra^2\int_0^1 w^*\theta\, dz = -Ra^2\int_0^1 \theta\left(D^2-a^2P^*\right)\theta^* dz, \tag{46}$$

$$-Ra^2\int_0^1 w^*\phi_1 dz = R_1a^2\tau_1\int_0^1 \phi_1\left(D^2-a^2-\frac{p^*}{\tau_1}\right)\phi_1^* dz, \tag{47}$$

$$-Ra^2\int_0^1 w^*\phi_2 dz = R_2a^2\tau_2\int_0^1 \phi_2\left(D^2-a^2-\frac{p^*}{\tau_2}\right)\phi_2^* dz. \tag{48}$$

Combining Eqs. (45) - (48), we get

$$\int_0^1 w^*(D^2-a^2)\left(D^2-a^2\frac{p}{\sigma}\right)wdz =$$

$$-Ra^2\int_0^1 \theta(D^2-a^2-p^*)\theta^* dz + R_1a^2\tau_1\int_0^1 \phi_1$$

$$\left(D^2-a^2-\frac{p^*}{\tau_2}\right)\phi_1^* dz + R_2a^2\tau_2$$

$$\int_0^1 \phi_2\left(D^2-a^2-\frac{p^*}{\tau_2}\right)\phi_2^* dz \tag{49}$$

Integrating the various terms of Eq. (49) by parts for an appropriate number of times and utilizing any of the boundary conditions (40)-(43), we obtain

$$\int_0^1 (|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2)$$

$$+\frac{p}{\sigma}\int_0^1 (|Dw|^2 + a^2|w|^2)dz = Ra^2$$

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2 + p^*|\theta|^2)dz - R_1a^2\tau_1$$

$$\int_0^1 \left(|D\phi_1|^2 + a^2|\phi_1|^2 + \frac{p^*}{\tau_1}|\phi_1|^2\right)dz - R_2a^2\tau_2$$

$$\int_0^1 \left(|D\phi_2|^2 + a^2|\phi_2|^2 + \frac{p^*}{\tau_2}|\phi_2|^2\right)dz \tag{50}$$

Equating the imaginary parts of both sides of Eq. (50) and cancelling pi ($p_i \neq 0$) throughout from the resulting equation, we have

$$\frac{1}{\sigma}\int_0^1 (|Dw|^2 + a^2|w|^2)dz = -Ra^2$$

$$\int_0^1 |\theta|^2\, dz + R_1a^2\int_0^1 |\phi_1|^2\, dz + R_2a^2\int_0^1 |\phi_1|^2\, dz \tag{51}$$

Now, from Eq. (38) we derive that

$$\int_0^1 \left(D^2-a^2\frac{p}{\tau_1}\right)\phi_1\left(D^2-a^2\frac{p^*}{\tau_1}\right)\phi_1^*$$

$$dz = \frac{1}{\tau_1^2}\int_0^1 |w|^2\, dz. \tag{52}$$

Integrating the various terms on the left hand side of Eq. (52) by parts for an appropriate number of times and making use of the boundary conditions on ϕ_1, it follows that

$$\int_0^1 (|D^2\phi_1|^2 + 2a^2|D\phi_1|^2 + a^4|\phi_1|^2)dz$$

$$+\frac{2p_r}{\tau_1}\int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)dz + \frac{|p|^2}{\tau_1^2}$$

$$\int_0^1 |\theta_1|^2\, dz = \frac{1}{\tau_1^2}\int_0^1 |w|^2\, dz \tag{53}$$

Since, $p_r \geq 0$, it follows from Eq. (53) that

$$\int_0^1 |\phi_1|^2\, dz \leq \frac{1}{|p|^2}\int_0^1 |w|^2\, dz \tag{54}$$

Similarly, from Eq. (39), by adopting the same procedure, we get

$$\int_0^1 |\phi_2|^2\, dz \leq \frac{1}{|p|^2}\int_0^1 |w|^2\, dz \tag{55}$$

Now making use of in equalities (54) and (55), we can write Eq. (51) as

$$\frac{1}{\sigma}\int_0^1 |Dw|^2\, dz + \frac{a^2}{\sigma}\int_0^1 \left[1 - \frac{(R_1+R_2)\sigma}{|p|^2}\right]|w|^2 dz$$

$$+\ Ra^2\int_0^1 |\theta|^2\, dz < 0, \tag{56}$$

which clearly implies that

$$|p|^2 < (R_1 + R_2)\sigma. \tag{57}$$

This establishes the desired result.

The above theorem may be stated in an equivalent form as: the complex growth rate of an arbitrary, neutral or unstable oscillatory perturbation of growing amplitude in a triply diffusive fluid layer heated from below, must lie inside a semicircle in the right half of the (p_r, p_i) – plane whose centre is at the origin and radius equals $\sqrt{(R_1+R_2)\sigma}$. Further, it is proved that this result is uniformly valid for quite general nature of the bounding surfaces.

Note: If we take $R_2 = 0$, it follows from the above theorem that we have the case of double diffusive convection and we obtain the result of Banerjee et al. (1981) as a consequence.

Theorem 2: If $R > 0$, $R_1 > 0$, $R_2 < 0$, $p_r \geq 0$ and $p_i \neq 0$ then a necessary condition for the existence of a nontrivial solution $(w, \theta, \phi_1, \phi_2, p)$ of Eqs. (36)-(39) together with boundary conditions (40) or (41) or (42) or (43) is that

$$|p|^2 < R_1\sigma.$$

Proof: Putting $R_2 = -|R_2|$ in Eq. (36) and adopting the procedure exactly similar to the one used in proving Theorem 1 we obtain the desired result.

The above theorem may be stated from the physical point of view as 'the complex growth rate (p_r, p_i) of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude, in a triply diffusive fluid layer with one of the components as heat, must lie inside a semicircle in the right- half of the (p_r, p_i)-plane whose centre is origin and radius equals $\sqrt{R_1\sigma}$, where R_1 is the Rayleigh number for the stabilizing concentration component (R_2 being negative disappear from the final result). Further, it is proved that above result is uniformly valid for quite general nature of the bounding surfaces.

3. Conclusions

A linear stability analysis is used to derive the upper bounds for complex growth rates in triply diffusive convection problem. These bounds are important especially when both the boundaries are not dynamically free so that exact solutions in the closed form are not obtainable. Further, the results so obtained are uniformly valid for all the combinations of rigid and free boundaries.

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References

Baines P G and Gill A E On thermohaline convection with linear gradient *J Fluid Mech* **37** (1969) 289-306

Banerjee M B, Katoch D C, Dube G S and Banerjee K Bounds for growth rate of perturbation in thermohaline convection *Proc Roy Soc London Ser A* **378** (1981) 301-304

Griffiths R W The influence of a third diffusing component upon the onset of convection *J Fluid Mech* **92** (1979) 659-670

Lopez A R, Romero L A and Pearlstein A J Effect of rigid boundaries on the onset of convective instability in a triply diffusive fluid layer *Physics of Fluids A* **2(6)** (1990) 897-902

Nield D A The thermohaline Rayleigh-Jeffreys problem *J Fluid Mech* **29** (1967) 545-558

Pearlstein A J, Harris R M and Terrones G The onset of convective instability in a triply diffusive fluid layer *J Fluid Mech* **202** (1989) 443-465

Rionero S Triple diffusive convection in porous media *Acta Mech* **224** (2013a) 447-458

Rionero S Multicomponent diffusive-convective fluid motions in porous layers ultimately boundedness, absence of subcritical instabilities, and global nonlinear stability for any number of salts *Phys Fluids* **25** (2013b) 054104(1-23)

Ryzhkov I I and Shevtsova V M On thermal diffusion and convection in multicomponent mixtures with application to the thermogravitational column *Physics of Fluids* **19** (2007) 027101 (1-17)

Ryzhkov I I and Shevtsova V M Long wave instability of a multicomponent fluid layer with the soret effect *Physics of Fluids* **21** (2009) 014102 (1-14)

Stern M E The salt fountain and thermohaline convection *Tellus* **12** (1960) 172-175

Turner J S The behaviour of a stable salinity gradient heated from below *J Fluid Mech* **33** (1968) 183-200

Turner J S Multicomponent convection *Ann Rev Fluid Mech* **17** (1985) 11-44

Veronis G On finite amplitude instability in thermohaline convection *J Mar Res* **23** (1965) 1-17.