Proc Indian Natn Sci Acad **79** No. 4 December 2013 Spl. Issue, Part A, pp. 679-685
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DOI: 10.16943/ptinsa/2013/v79i4/47999

Research Paper

Investigation of External Inversion of Thin-Walled Tubes

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(Received 25 December 2012; Accepted 05 February 2013)

In this paper a computational study of development of external inversion of round metallic aluminium tubes over a fixed profile die is presented. Inversion mode of deformation is analyzed in detail by using a finite element code FORGE2. The proposed finite element model for this purpose idealizes the deformation as axisymmetric. Six nodded triangular elements are used to descretize the domain. The material is modelled as rigid-viscoplastic. Typical variations of equivalent strain rate and equivalent strain and energy dissipation mechanisms during inversion are presented. Results presented are of help in understanding the mechanics of the inversion process. A few predicted results which include the geometry of inverted tube, load-compression variation during inversion process are compared with their experimental counterparts to validate the computational model.

Key Words: External Inversion; FORGE2; Axisymmetric Deformation; Large Deformation

1. Introduction

Energy absorbing devices are employed at the places where collision may cause very serious destructuion; which includes loss of human life and other different important parts of the structure. Thin and thick walled metallic tubes are very efficient in absorbing the impact energy and consequently widely used as structural component in energy absorbing devices [1-2]. During the impact process these tubular elements deform plastically in different modes of collapse namely axisymmetric concertina, axisymmetric mulptiple barrelling [1], nonsymmetric diamond, transverse flattening of tubes [2] and axisymmetric external inversion [3-4]. These modes of collapse depend upon the end conditions of the element, cross-sectional shape of element and direction of loading. Inversion is the mode of collapse which develops when a round tube is compressed between a flat platen at one end and a radiused die at other [3]. Inversion mode of deformation may be used to manufacture a double walled tube that is difficult to be manufactured by any other technique. Therefore the analysis of external inversion process is equally important for manufacturing technology as well as for impact and crashworthiness studies.

Kinkead [5] presented the analysis of internal inversion of a round tube. Analytical results were compared with existing experimental results. Effects of strain hardening of the tube material and friction at the die-tube interface were studied. Change in wall thickness of the inverted tube was also presented. Chirwa [6] presented the investigation of plastic collapse of a tapered thin walled metal tube called as an “inverbucktube” (Inverbuck is abbreviation for inversion and buckling). An approximate analysis of the collapse mechanism was suggested. Predicted specific energy, forming load and mode of collapse were found to be in good agreement with experimental

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results. Reid *et al*. [7] studied the external and internal inversion of round metallic tubes by conducting experiments and analysis of the inversion process. Effect of the die radius on external inversion was investigated [8]. Splitting of tubes and influence of stopper plates on load compression curve and mode of deformation were also investigated [7]. They also employed the finite element code ABAQUS to the study internal inversion of round tubes. Development of nosing and internal inversion were also discussed. Miscow and Al-qureshi [9] predicted the dynamic inversion load based on the quasi-static data and the law of conservation of energy.

In this paper a computational investigations of the external inversion of round tubes over fixed profile dies is presented. Non-linear finite element code FORGE2 is employed to carry out the computations. A simulation model of the inversion process is presented and the external inversion process is studied. Effects of geometrical parameters of tube and radius of die are studied. The variation of different components of stress, strain and strain rate during the deformation process at different stages of inversion are presented, analyzed and discussed. Understanding of material flow during the inversion process and ultimately the development of double walled inverted tube on the basis of the computational findings is presented and discussed. A few experimental results are also presented to mainly validate the proposed simulation model and also support the computational findings. Fig. 1(a) shows the schematic diagram of the external inversion of round tube used in study.

(a) **(b)**

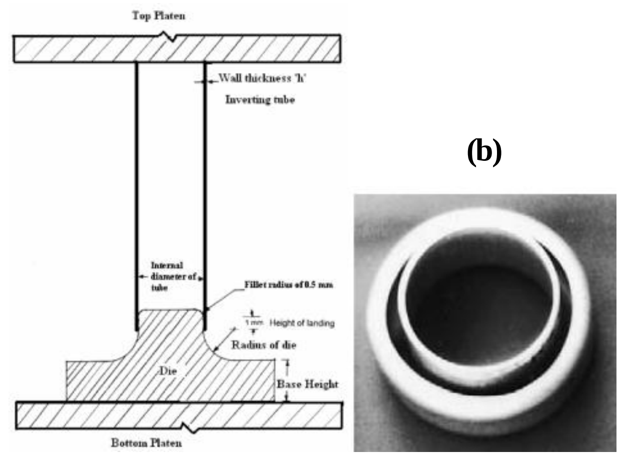


Fig. 1: (a) Experimental set-up used for performing external inversion of different round tubes; (b) Photographic view of typical inverted tube

2. Experimental Investigation

A Universal Testing Machine INSTRON (Model No.

Table 1: Comparison between FEM simulation results and experimental results of inversion mode of collapse

Sp. no.	r(mm)	Geometrical properties of tested tubes				Mode of deformation	Value of b_e (mm)		Delta/r	
		D (mm)	h (mm)	OD (mm)	D/h		Exp.	Comp.	Exp.	Comp.
INV383	3	37.04	1.12	38.16	33.07	Inversion	50.68	51.35	1.230	1.286
INV385	5	37.04	1.12	38.16	33.07	Inversion	52.42	56.2	0.825	1.014
INV387	7	37.04	1.12	38.16	33.07	Inversion + cracking	The tube was cracked			
INV5013	3	48.92	1.68	50.6	29.12	Inversion + concertina folding	60.24	62.6	1.083	1.280
INV5015	5	48.92	1.68	50.6	29.12	Inversion	65.84	69.2	0.930	1.098
INV5017	7	48.92	1.68	50.6	29.12	Inversion	70.42	72.44	0.828	0.900
INV5023	3	48.7	1.3	50	37.46	Inversion + concertina folding	63.44	65.16	1.337	1.480
INV5015	5	48.7	1.3	50	37.46	Inversion	68.02	70	1.031	1.130
INV5017	7	48.7	1.3	50	37.46	Inversion	72.84	74.42	0.909	0.965

r- Die radius; D- Average diameter; h- Wall thickness; OD- Outer diameter; Exp.- Experimental; Comp.- Computed; Delta is radius of inverted tube equivalent to die radius = $(b_e\text{-}D+h)/4$, which represents the diameter of knuckle; Relative diameter of knuckle = 4(delta-h)/OD

1197) of 50 T capacity was employed for experimentation. Aluminium tubes of round section having different geometrical properties (see Table 1) were inverted over dies of radii r = 3 mm, 5 mm and 7 mm (see Fig. 1). It was observed in experimentation that apart from the inversion, two other modes of deformations were also triggered after initial inversion of tube. Circumferential crack was initiated in the specimen INV387 after 12 mm compression value (δ). In the specimen INV5013 after 18.2 mm compression value (δ) development of an axisymmetrical concertina ring started near the top platen. For specimen INV5023 also after initial inversion of tube an axisymmetrical concertina ring started developing near the inverting die. Remaining all other specimens were successfully inverted and the inversion process was completed. Photographic view of a typical inverted tube is shown in Fig. 1(b). It is clear that if the inversion process continue successfully then it can be idealized as a two-dimensional axisymmetric deformation. Fig. 2(a) shows the deformed shapes of some of the representative specimens during inversion process. Their corresponding load-compression curves are also shown in Fig. 2(b).

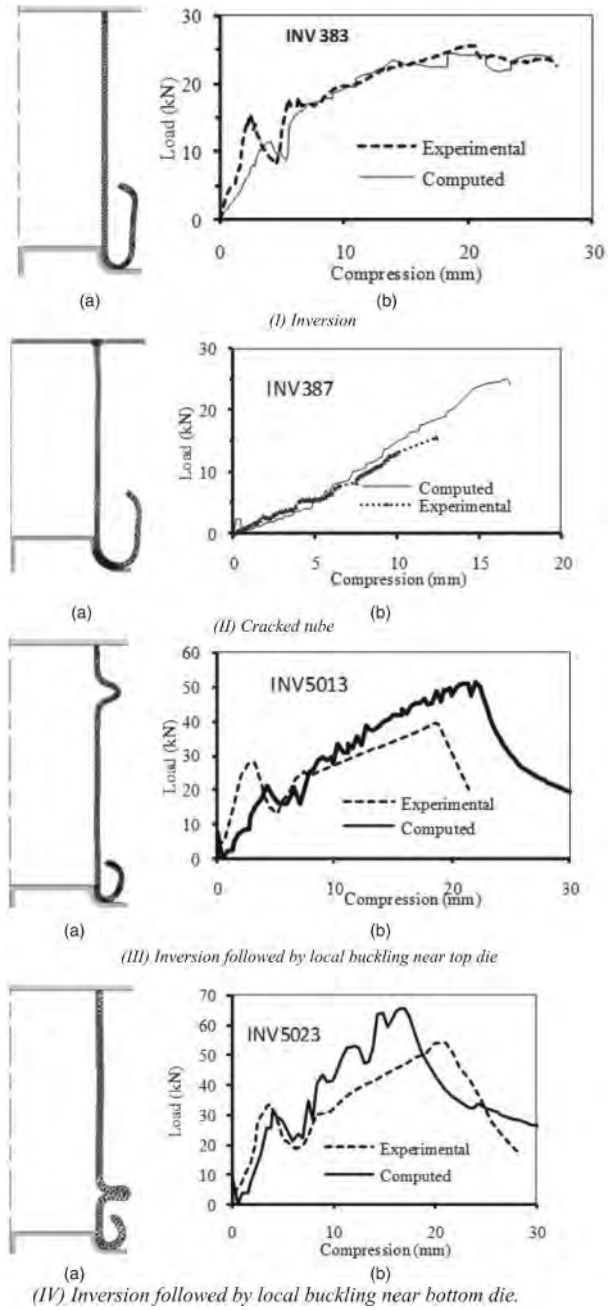


Fig. 2: Different (a): modes of deformation with their corresponding; (b): load-compression variations, which may occur during the external inversion of thin-walled tubes over a die

3. Finite Element Simulation

Inversion of a round tube over a cylindrical die is a case of axisymmetric deformation (see Fig. 1(b)). In the proposed model of the process, the material is assumed as homogeneous, isotropic, incompressible and rigid visco-plastic. The details of the formulation and solution technique can be found elsewhere [10]. The model has been analysed with the help of a Finite Element code FORGE2 [11]. The FORGE2 code is based on the flow formulation in which the deforming material obeys the following constitutive law

$$\bar{\sigma} = K_1 \left(1+a\bar{\varepsilon}\right) \dot{\bar{\varepsilon}}^{m} \tag{1}$$

where $K_1 = K_0 (\sqrt{3})^{m+1}$, $\bar{\sigma}$, is equivalent stress, $\bar{\varepsilon}$ is equivalent strain, $\dot{\bar{\varepsilon}}$, is equivalent strain rate, K_1 is consistency of material, K_0 is a constant, a is the strain hardening parameter, m is the strain rate sensitivity index.

The frictional stress at the tube and platen interfaces is assumed to obey the Norton-Hoff frictional law [12]. Accordingly the frictional stress (τ_f) is assumed to be governed by the following relation

$$\tau_f = \alpha K \tilde{v}_f^{p} \tag{2}$$

where $\tilde{v}_f$ is the sliding velocity between the platen/die and the deforming tube, α is friction factor and p is exponent of sliding velocity. Contact between the tube and top platen, and between the tube and the die is modeled as sliding unilateral [11]. Friction factor at the platen-tube and tube-die interface is assumed to be given. The platen and the die are assumed as rigid. The die remains stationary during the process whereas the platen is imparted a constant downward velocity of 1mm/min. Six noded isoparametric triangular elements are used for discretising the problem domain. Fig. 3(a) shows the proposed finite element model used for computational studies. For analysis purpose, the total compression process is divided into a large number of small steps. The FORGE2 code is employed for analyzing the computational model. Number of elements used for discretising the problem domain of different specimens are varied from 2400 to 2700. Approximately five remeshings were required to simulate the complete inversion process.

To perform the finite element simulation of the inversion process material property parameters namely strain hardening coefficient (a), strain rate sensitivity index (m) and consistency of material (K_0) are required as input data. These were evaluated by carrying out uniaxial tension tests on standard tension specimens at different strain rates.

4. Verification of Computational Model

Fig. 2(a) depicts typical computed deformed profiles of the tube specimens INV383, INV387, INV5013 and INV5023 to represent the different types of mode of deformation occurred during inversion process. The true and computed mode of deformation of different tubes are almost identical. To facilitate comparison between observed and predicted deformed profiles of inverted tubes, one characteristic dimensions namely b_e is defined (Fig. 3(b)). The parameter b_e represents the external diameter of the initially formed knuckle or finally inverted tube on reaching steady state. Comparison of theoretical and computed values of b_e is given in Table 1. The comparison of corresponding load-compression curves are shown in Figs. 2(b). It can be concluded from Fig. 2 and Table 1 that the computational model predicts the mode of deformation as well as load-compression variations fairly well.

5. Typical Computational Results

Simulation results of inversion process over dies of radii $r = 3$ mm of a round aluminium tube having mean diameter $D = 37.04$ mm, wall thickness $h = 1.12$ mm, length = 80.4 mm are presented as a representative case to understand the development of equivalent strain rate and equivalent strain to help in understanding the inversion process. The material property parameters for this tube are $K_o = 176.5$ Mpa, $a = 1.4$, and $m = 0.0107$.

To assist discussion, the typical profile of a deformed tube (Fig. 3(b)) may be divided into four zones. The four zones are characterized in the following way:

Zone I: Undeformed portion of the tube

Zone II: Portion of tube below zone I and covers the whole quarter circular part of die.

Zone III: Portion of tube starts from the end of the curvature of the die and extends upto the fully developed knuckle.

Zone IV: Upper top curved portion of fully developed knuckle which originats from the zone III after reaching steady state of inversion load. In this zone deformation does not occur.

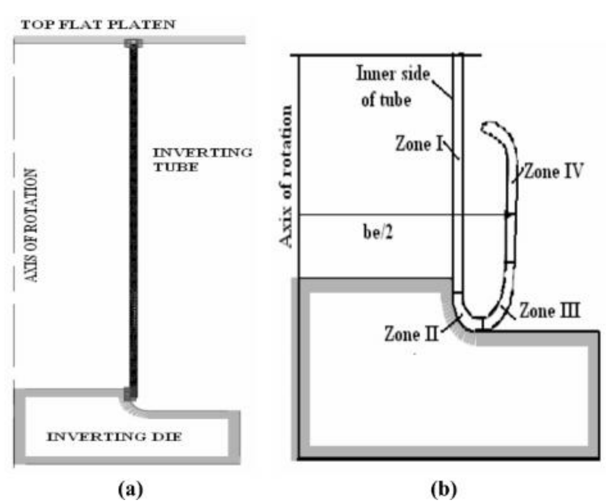
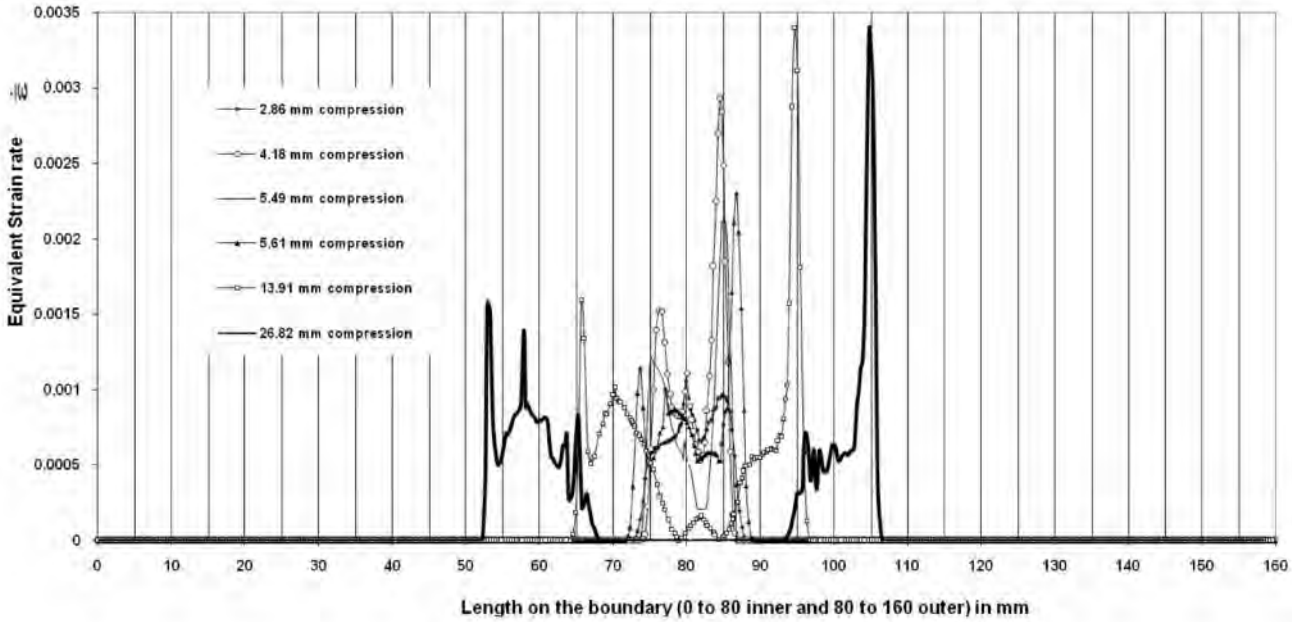
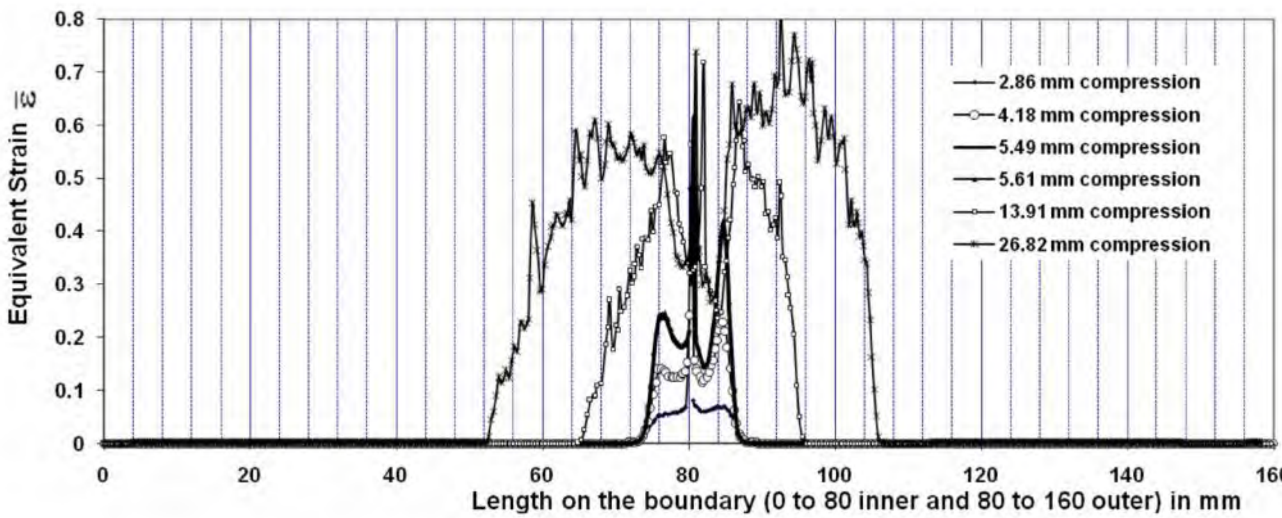


Fig. 3: (a): Proposed Computational Model, (b): Division of deformed profile into four zones of interest



(a) *Equivalent strain rate*



(b) *Equivalent strain*

Fig. 4(a-b): Variations of (a): equivalent strain rate (/s) and (b): equivalent strain on outer boundary and inner boundaries of the tube during inversion process of specimen INV383

Figs. 4(a) and 4(b) show the variations of the equivalent strain rate and equivalent strain on the outer boundary and inner boundary of the tube at different compression values (δ). In these graphs, the X-axis represents the length of the tube measured from the inner side of the tube from the contact point of the top platen and towards the die and back to top platen contact point. The point corresponding to 80.4 mm is the demarcation point between inner and outer boundary. At the beginning of the compression process the seat of the higher equivalent strain rate ($\dot{\bar{\varepsilon}}$) remains around the tube-die contact region of the tube and as the compression process proceeds it moves in the new virgin tube. The values of the equivalent strain are higher in outer boundary of the tube as compared to their inner boundary counterpart. This may be due to the higher hoop strain rate development on outer boundary as compared to inner boundary. It is clear from the Fig. 4(a) that, at any value of compression, equivalent strain rate ($\dot{\bar{\varepsilon}}$) applies only over a certain length of the tube in which actually the deformation currently occurs. In other words in the remaining portion of the tube, the values of $\dot{\bar{\varepsilon}}$ are negligible. The length of this segment of the tube increases with increase in compression value ($\hat{\delta}$) and attains a threshold value equal to 13.91 mm when complete knuckle develops. It can also be observed that throughout the compression process the highest value of the equivalent strain rate ($\dot{\bar{\varepsilon}}$) occurs on the die-tube contact point. This die-tube contact point changes its location during inversion process. During initial inversion it falls on inner boundary while after that on outer boundary of the tube.

From Fig. 4(b) it is clear that throughout the inversion process, equivalent strains ($\bar{\varepsilon}$) developed on outer boundary are of higher values as compared to the equivalent strains on inner boundary at any compression value. Upto 5.61 mm compression the equivalent strain $\bar{\varepsilon}$ develops only in that tube portion which is in contact with the die. Highest value of the equivalent strain ($\bar{\varepsilon}$) upto 5.61 mm compression is 0.61 and it occurs at the toe of the tube which falls on the inner boundary of the tube and in zone II.

As the inversion process proceeds the equivalent strain ($\bar{\varepsilon}$) starts developing in the new virgin portion of the tube which comes in contact with the die and as a result location of the highest value of equivalent strain ($\bar{\varepsilon}$) shifted to some other point which falls in zone II. This development gets continue till the knuckle fully develops and this happens at 13.91 mm compression value. After 5.61 mm compression value toe portion of the tube depart from the die, so origination of zone III starts. Consequently a knuckle type shape forms and get fully developed upto 13.91 mm compression value. By comparing the variation of equivalent strain ($\bar{\varepsilon}$) at 13.91mm, 20mm and 26.82 mm it can be concluded that the variation is of similar type on both the boundaries but it occurs in different portions of the tube. This shows that after 13.91 mm compression value the inversion load reaches to a steady value, so the highest value of the equivalent strain ($\bar{\varepsilon}$) is also almost remain unchanged.

The distributions of equivalent strain rate ($\dot{\bar{\varepsilon}}$) and equivalent strain ($\bar{\varepsilon}$) show that throughout the inversion process tube in zone II deforms continuously while tube in zone I remains idle and zone III originates from zone II and zone IV from zone III. During the inversion process with increase in compression value highest value of equivalent strain ($\bar{\varepsilon}$) also increases in the tube only upto 13.91 mm compression value.

The following points can be concluded on the basis of the typical computational results and their discussion;

1. The value of the equivalent strain at different outer and inner locations of the tube is not equal in magnitude throughout the compression process. This indicates that the tube is subjected to bending and compression or tension.

2. In the whole inversion process four zones I, II, III and IV exist. With progress of inversion process the areas of zone III increases, I decreases and II remains unchanged. Zone IV develops after complete formation of knuckle and it originates from zone III.

3. Tube is deformed and consequently inverted mainly due to the hoop stretching combined with the meridional bending in initial stage and rolling over the die.

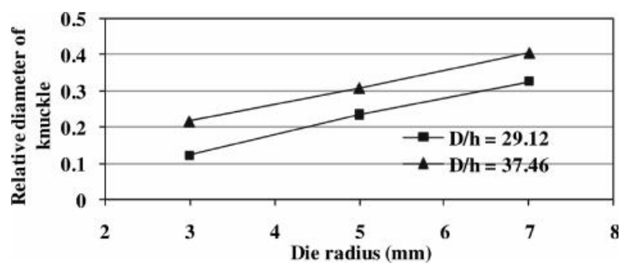


Fig. 5: Variation of relative diameter of knuckle with radius of die for different specimens

Fig. 5 presents the variation of relative diameter of knuckle for different specimens having different D/h ratios inverted over dies of different radius. It is clear from this Fig. that as D/h ratio increases the relative diameter of knuckle also increases for the same die radius. Moreover, for the same tube as die radius increases the relative diameter of knuckle also increases. This is due to the conformity of the tube over the die of higher radius.

6. Conclusions

In this paper development of external inversion mode of deformation of round tubes using Finite Element Analysis is presented. Experiments are also conducted on round aluminium tubes of different aspect (i. e. diameter/thickness) ratios varying from 29 to 38. The tubes were axially compressed at a rate of 1mm/ minute on an universal testing machine INSTRON. The tubes were placed between the top flat platen and a radiused die resting on the bottom platen of the machine to create external inversion mode of deformation.

The computed results are compared with their experimental counterparts to validate the computational model. Effect of radius of the inverting die on the inversion process is discussed. On the basis of the computational and experimental findings mechanics of the inversion process is presented and discussed. It is seen that during inversion of tube, the flow of material does not fully conform to the die profile during the non-steady portion of the process. This is contrary to the assumption made in the past by different researchers in proposing analytical models [5, 9]. The diameter of the knuckle is dependent on the radius of the die.

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