

Research Paper

Geometrically Nonlinear Analysis of Thin-Walled Structures using Efficient SPH Shell-Based Method

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In this paper a meshless shell-based formulation is proposed for the geometrically nonlinear analysis of thin-walled structures using an explicit dynamic scheme based on the Smoothed Particle Hydrodynamics (SPH) method. In the present investigation the SPH method is modified to deal with shell-like structures, while keeping its character of a strong formulation based on the principle of collocation directly applied on the differential equilibrium equations. The current SPH formulation is an extension of the continuum-improved and stabilized SPH method allowing a thin structure to be modelled using only one layer of particles to represent the shell mid-surface. Application of the present SPH shell-based formulation for the analysis of several benchmarks including geometrically nonlinear behaviour shows its validity and its potential especially in terms of CPU time saving while keeping a very good level of accuracy compared to the classical SPH method and to the Finite Element method.

Key Words: Geometrically Nonlinear Analysis; Smoothed Particle Hydrodynamics; Meshless; Mindlin-Reissner Shell; Explicit Dynamics; Strong Formulation

1. Introduction

Since its creation in the 1950s, the Finite Element Method (FEM) has become extensively used. An outstanding feature of the FEM consists in splitting a structure into a discrete number of elements with basic features which makes the solution easy for approximation. Nevertheless in some problems involving high gradients such as those encountered in crashworthiness or in blast simulations; the FEM may diverge unless a very fine mesh is used and consequently a very small time step is required, which makes this method computationally very expensive and not useful in practical situations.

Unlike the conventional FEM, one of the most efficient meshless methods is the Smoothed Particle Hydrodynamics (SPH) method [1, 2] which is based

on the principle of collocation directly applied on the strong formulation of the problem. Through the literature [3-5], it has been established that the SPH method is more competitive than the FEM in the case of large distortions or propagation of cracks with arbitrary and complex paths [5].

Though, the classical SPH method formulated using a Lagrangian Formulation suffers from several imperfections such as the lack of consistency, the tensile instability and the emergence of zero energy modes in the numerical computation [6-8]. This investigation introduces a Corrective Smoothed Particle Hydrodynamics (CSPH) method [9] to restore the consistency by reproducing the kernel functions. We also employ a Total Lagrangian Formulation with respect to the initial undeformed configuration to reduce the tensile instability problem [10]. Also in

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order to avoid oscillations of the numerical solution especially in the shocking regions, an artificial viscosity function is introduced.

From the literature review, the classical SPH method has been developed and successfully used in structural mechanics as well as in the modelling of forming processes and in metal cutting using only three-dimensional (3-D) continuum approach [11, 12]. However modelling thin structures using a 3-D continuum approach is computationally very expensive, because several particles need to be placed in the thickness-direction. It has been shown [12] that a minimum of three particles is needed in elasticity and five particles for plasticity are necessary to ensure a good quality of results. Meshless shell formulation with a single layer of particles on the mid-plane has attracted numbers of researchers. For instance, Krysl and Belytschko [13] combined the Kirchhoff plate theory with Element Free Galerkin method for solving structural problems involving plates.

In this work, a new SPH shell-based formulation is proposed and validated for the geometrically nonlinear analysis of thin-walled structures using the explicit dynamic scheme. The proposed SPH shell-based formulation is established using the Mindlin–Reissner's theory which accounts for transverse shearing stresses. The formulation is an extension of the Continuum-Corrected and stabilized SPH method allowing a thin structure to be modeled using only one layer of particles characterizing the mean position of the shell mid-surface.

Several numerical applications are treated where a nonlinear analysis is conducted using the developed explicit dynamic SPH shell-based formulation, to study the geometrically nonlinear behavior of thin-walled structures undergoing large displacements and large rotations. Comparison of numerous results obtained to the analytical reference or to FEM numerical results obtained using ABAQUS[®] Explicit Dynamic code; show that the SPH shell-based formulation may be a new alternative to the FEM. The developed procedure shows also its validity and its potential especially in terms of CPU time saving while keeping a very good level of

accuracy compared to the classical SPH method and to the Finite Element method.

2. The SPH Method using Total Lagrangian Formulation

Smoothed Particle Hydrodynamics (SPH) employs a set of finite number of particles to represent the state of a system and approximates the field variables through kernel estimates. In this method, the integral representation of a field f at each particle i is given through its neighbouring nodal parameters,

$$f_i = \sum_{j=1}^N f_j \cdot V_j W(r_{ij}, h) = \sum_{j=1}^N f_j \cdot \phi_{ij} \quad (1)$$

with

$$W(q, h) = \begin{cases} \frac{10}{7\pi h^2} \left(1 - \frac{3}{2}q^2 + \frac{3}{4}q^3 \right) & \text{if } 0 \leq q < 1 \\ \frac{5}{14\pi h^2} (2 - q)^3 & \text{if } 1 \leq q < 2 \\ 0 & \text{if } q \geq 2 \end{cases}$$

where N is the number of neighbouring particles, j is a neighbouring particle for which the SPH shape function ϕ_{ij} is not zero and its volume V_j . $W(q, h)$ is the kernel function based on the the most widely used cubic spline function [2]. $q = \|\hat{x}_{ij}\| / h$, where $\|\hat{x}_{ij}\| = \|\hat{x}_{ij} - \hat{x}_j\|$ represents the distance between particles i and j , and h is the smoothing length defining the size of the support domain.

In meshless-based methods, the gradient of the field f can be approximated by

$$\nabla f_i = \sum_{j=1}^N f_j \cdot \nabla \phi_{ij} \text{ or } \nabla f_i = \sum_{j=1}^N (f_j - f_i) \cdot \nabla \phi_{ij} \quad (2)$$

The second part of Eq. (2) is preferred because it can ensure that the derivative of a constant field can be approximated correctly; i.e. zero-th completeness condition. However, still first-order completeness not achieved on the domain's boundaries. To fulfill that, we adopt the so-called "Corrected SPH" (CSPH) method [9] to normalize the derivatives of the shape function.

$$\nabla f_i = \mathbf{B} \cdot \sum_{j=1}^N (f_j - f_i) \cdot \nabla \phi_{ij} = \sum_{j=1}^N (f_j - f_i) \cdot \nabla \phi_{ij}$$

with

$$\mathbf{B} = \left[\sum_{j=1}^N (x_j - x_i) \otimes \nabla \phi_{ij} \right]^{-1} \quad (3)$$

In solid and structural mechanics, the continuity and momentum conservation equations for a particle i can be estimated by

$$\left(\frac{\partial \rho}{\partial t} \right)_i = -\rho_i \cdot \text{div}(\mathbf{v}) = -\rho_i \sum_{j=1}^N (\mathbf{v}_j - \mathbf{v}_i) \cdot \nabla \phi_{ij} \quad (4)$$

$$\ddot{u}_i = \left(\frac{\partial \rho}{\partial t} \right)_i = \text{div}(\boldsymbol{\sigma}) / \rho = \sum_{j=1}^N \rho_j \left(\frac{\sigma_i}{\rho_i^2} + \frac{\sigma_j}{\rho_j^2} + \Pi_{ij} \right) \cdot \nabla \phi_{ij} \quad (5)$$

Following [1, 2] we introduced a scalar Π_{ij} as a term of the artificial viscosity allowing the algorithm to be capable of modelling shock waves problems, where more technical details and information can be found in reference [2].

These frequently-used equations are intrinsic Eulerian formulations expressed in spatial coordinates, which can lead to tensile instabilities usually encountered in shock applications [10]. It also showed that the use of Lagrangian kernel function can reduce this problem. In the total Lagrangian formulation, the initial configuration is defined as a reference configuration, therefore the initial density and volume are used in Eq. (4); so that it is not necessary to update these quantities. In the momentum conservation Eq. (5), the Cauchy stress should be replaced by the first Piola-Kirchhoff stress,

$$\begin{aligned} \ddot{u}_i &= \left(\frac{\partial \mathbf{v}}{\partial t} \right)_i = \text{DIV}(\mathbf{P}) / \rho_0 \\ &= \sum_{j=1}^N \rho_{0j} \left(\frac{P_i}{\rho_{0i}^2} + \frac{P_j}{\rho_{0j}^2} + \Pi_{0ij} \right) \cdot \nabla_0 \Phi_{0ij} \end{aligned} \quad (6)$$

3. The CSPH Shell-based Formulation

The continuum CSPH formulation will be extended to be able to model thin structures according to the Mindlin-Reissner thick shell theory which accounts

for transverse shearing stresses. Then only one layer of particles located on the mid-surface of the shell structure can be used to represent the entire shell behaviour. Each particle has five degrees of freedom (three translations and two rotations expressed into the local framework tangent to the shell mid-surface).

In the shell structure with a thickness of t , considering any point q located at a distance ξ from the shell mid-surface, the position vector can be expressed as

$$\tilde{\mathbf{x}} = \tilde{\mathbf{x}}_m + \xi \tilde{\mathbf{n}} \quad \xi \in \{-t/2, t/2\} \quad (7)$$

where $\tilde{\mathbf{n}}$ is the pseudo-normal vector.

The initial local coordinate system showed in Fig. 1 can be transformed to the initial global cartesian system using a transformation matrix $\mathbf{Q}_{L0}$, and the displacement vector can be expressed as follows:

$$\begin{aligned} \tilde{\mathbf{X}}_{L0} &= \mathbf{Q}_{L0} \tilde{\mathbf{X}}_0 \quad \text{and} \quad \tilde{\mathbf{u}}(\tilde{\mathbf{X}}_{L0}) = \tilde{\mathbf{u}}_m(\mathbf{X}_{L0}, \mathbf{Y}_{L0}) \\ &+ \mathbf{Z}_{L0} [\tilde{\mathbf{n}}(\mathbf{X}_{L0}, \mathbf{Y}_{L0}) - \tilde{\mathbf{n}}_0(\mathbf{X}_{L0}, \mathbf{Y}_{L0})] \end{aligned} \quad (8)$$

In the SPH method, the kernel function f is always expressed in the local framework system, therefore we use the same matrix $\mathbf{Q}_{L0}$ to evaluate its gradients directly in the global cartesian system as

$$\tilde{\nabla}_0 f = \mathbf{Q}_{L0} \tilde{\nabla}_{L0} f \quad (9)$$

Based on the previous equations, the deformation gradient tensor $\mathbf{F}$ and the Green-Lagrange Strain tensor $\mathbf{E}$ can be calculated and

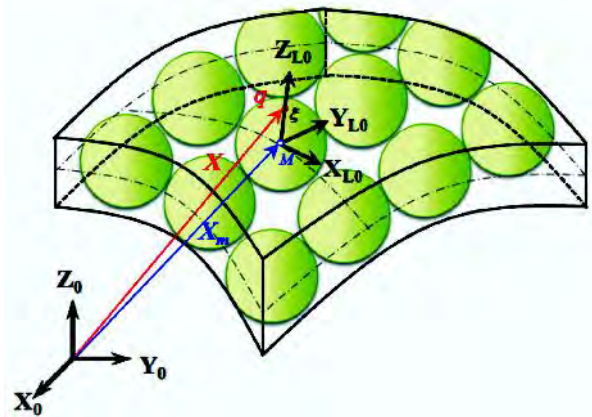


Fig. 1: Geometry of a shell modelled using SPH particles

expressed in the global cartesian system. In shell formulation, it is useful to split the Green-Lagrange Strain tensor $\mathbf{E}$ in two parts : a constant tensor in ξ through the thickness, called $\mathbf{E}_m$ associated to membrane and shearing effects, and a linear tensor in ξ through the thickness, called $\mathbf{E}_f$ which is associated to the bending effects.

To calculate the stresses based on the plane stress assumption, the Green-Lagrange strain tensor should be transformed into the local framework system of the current configuration. The transformation between the two coordinate systems is carried out using matrix $\mathbf{Q}_L$. The transformed membrane/shear and bending strain tensor may be obtained as follows

$$\begin{aligned}\varepsilon_{Lm} &= \mathbf{Q}_L \cdot \mathbf{F}^{-T} \cdot \mathbf{E}_m \cdot \mathbf{F}^{-1} \cdot \mathbf{Q}_L^T \quad \text{and} \\ \varepsilon_{Lf} &= \mathbf{Q}_L \cdot \mathbf{F}^{-T} \cdot \mathbf{E}_f \cdot \mathbf{F}^{-1} \cdot \mathbf{Q}_L^T\end{aligned}\quad (10)$$

In the case of elastic material, the in-plane stress related to the membrane σ_{Lm} , the bending σ_{Lf} and the transverse shear stress σ_{Ls} can be calculated according to the classical Hooke's material law. Integrating these relations through the thickness $\xi \in \{-t/2, t/2\}$, the membrane and shear resultant efforts $\mathbf{N}_{ij}$ and $\mathbf{T}_i$ and the bending moments $\mathbf{M}_{ij}$ can be obtained explicitly.

The dynamic equilibrium equations involving the internal force vector in the global system are evaluated according to

$$t\rho_0\ddot{u} = \text{DIV}(\mathbf{P}) \quad \text{with } \mathbf{P} = \det(\mathbf{F})\mathbf{Q}_L^T\mathbf{N}\mathbf{Q}_L\mathbf{F}^{-T}$$

and

$$\mathbf{N} = \begin{bmatrix} N_{xx} & N_{xy} & T_x \\ N_{yx} & N_{yy} & T_y \\ T_x & T_y & 0 \end{bmatrix} \quad (11)$$

where $\mathbf{P}$ is the first Piola-Kirchhoff (PK1) stress tensor, while $\mathbf{N}$ matrix includes the generalized membrane and shear resultant efforts conjugated to the PK2 stress.

The momentum equilibrium equations in the current final local system can be derived as follows

$$\begin{aligned}tI_x\ddot{\theta}_{xL} &= M_{xy,xL} + M_{yy,yL} + T_y, \\ tI_y\ddot{\theta}_{yL} &= M_{xx,xL} + M_{xy,xL} - T_x\end{aligned}\quad (12)$$

where I_x, I_y are rotation inertias, M_{ij} the bending moment components and $\ddot{\theta}_{xL}, \ddot{\theta}_{yL}$ the local angular accelerations.

Eq. (12) can be reorganized and written in a more convenient matrix form as

$$t\mathbf{I}\ddot{\theta}_L = \mathbf{L}\text{div}_L(\mathbf{M}) + \bar{T}_L$$

with

$$\begin{aligned}\mathbf{L} &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{M} = \begin{bmatrix} M_{xx} & M_{xy} & 0 \\ M_{yx} & M_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \bar{T}_L &= \begin{bmatrix} T_y \\ -T_x \\ 0 \end{bmatrix}\end{aligned}\quad (13)$$

Then a final transformation of Eq. (13) into the global coordinate system is needed, therefore the final expression can be given by

$$tI_0\ddot{\theta} = \text{DIV}(\mathbf{M}_0) + \bar{T}_0$$

with

$$\mathbf{M}_0 = \det(\mathbf{F})\mathbf{Q}_L^T\mathbf{L}\mathbf{M}\mathbf{Q}_L\mathbf{F}^{-T}; \quad \bar{T}_0 = \det(\mathbf{F})\mathbf{Q}_L^T\bar{T}_L \quad (14)$$

Using the explicit time integration scheme, the increment of displacement vector $\Delta\bar{u}$ and the increment of rotation vector $\Delta\bar{\theta}$ can be updated. Nevertheless by noticing that the angular velocities $\bar{w}$ at different time steps do not belong to the same tangent space and hence cannot be additive directly; then it is necessary to introduce a matrix $\mathbf{S}$ in order to transform vector $\bar{w}$ at the last time step into the current tangent space [18].

$$\begin{aligned}\bar{w}_{I(n+1)} &= \mathbf{S}(\Delta\bar{\theta})(\bar{w}_{I(n)} + \Delta t\ddot{\theta}_{I(n)}) \quad \text{defined } \mathbf{S}(\bar{\theta}) \\ &= \frac{\sin\theta}{\theta}\mathbf{I} + \frac{1-\cos\theta}{\theta^2}\hat{\Theta} + \frac{\theta-\sin\theta}{\theta^3}\bar{\theta} \times \bar{\theta}\end{aligned}\quad (15)$$

where $\theta = \|\bar{\theta}\|$ and $\hat{\Theta}$ is the skew-symmetric tensor for which $\bar{\theta}$ is the axial vector.

The Rodrigues formula [18] associated with the increment of rotation vector $\Delta \hat{\theta}^{\mathcal{E}}$ is used to update the pseudo-normal vector $\hat{n}^{\mathcal{E}}$,

$$\hat{n}_{t(n+1)}^{\mathcal{E}} = \Delta \mathbf{R}(\Delta \hat{\theta}^{\mathcal{E}}) \cdot \hat{n}_{t(n)}^{\mathcal{E}};$$

with

$$\mathbf{R}(\hat{\theta}^{\mathcal{E}}) = \cos \theta \mathbf{I} + \frac{\sin \theta}{\theta} \hat{\Theta} + \frac{1 - \cos \theta}{\theta^2} \hat{\Theta} \times \hat{\Theta} \quad (16)$$

To avoid irregular behaviour of the pseudo-normal vectors of particles especially on the edges, each normal needs to be regularized using a special filter [7].

$$\hat{n}_i' = \hat{n}_i + \alpha \left(\frac{\sum_{j=1}^N (\hat{n}_j \phi_{ij})}{\sum_{j=1}^N \phi_{ij}} - \hat{n}_i \right) \quad (17)$$

where $\hat{n}_i'$ is the equivalent pseudo-normal and α is a regularization parameter which is fixed to 0.01 in this study, this value has been found to be suitable through all our numerical applications.

4. Numerical Applications

In this section, several numerical applications will be treated using the present CSPH shell-based formulation together with the explicit dynamic time integration scheme; in order to show its capabilities and prove its efficiency compared to the classical FEM.

4.1 Nonlinear Analysis of a Plate Buckling

The first application deals with a rectangular plate buckling using a geometrically nonlinear analysis. This interest of this example is the existence of an analytical solution which will allow evaluating precisely the efficiency of the CSPH shell-based method. The studied plate is simply supported on its four edges and submitted to an equal uniform compression load P on two opposite edges b as shown in Fig. 2. Due to the problem symmetry; only a quarter of the plate is modelled and discretized using only one layer of 25 x 25 particles through the plate mid-surface.

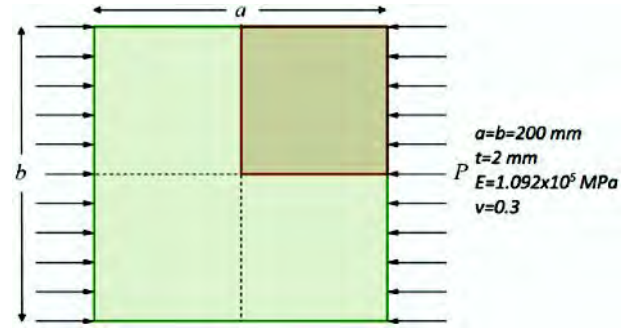


Fig. 2: Geometry and material characteristics of the plate

The analytic solution of the critical buckling load can be retrieved in [16], it is given by: $P_{cr} = \pi^2 E t^3 / 3a^2(1-\nu^2)$ which corresponds numerically to $P_{cr} = 78.96$ N/mm.

The geometrically nonlinear analysis is carried out using the plate edge displacement control method, therefore different horizontal edge displacements are applied until a maximal load $P = 100$ N/mm is achieved. In order to compare the CSPH solution to the FEM result, the plate is modeled using ABAQUS[®] code with 25 x 25_S4R shell elements. The central plate load-displacement curve is then extracted and a comparison is made between the different solutions as shown in Fig. 3.

As we can observe from Fig. 3, the SPH shell-based solution is in a very good agreement with FEM solution obtained using ABAQUS[®] code. The critical buckling load obtained using the geometrically nonlinear analysis based on SPH shell formulation is $P_{cr} = 75.5$ N/mm, with a small error of 4.38% with

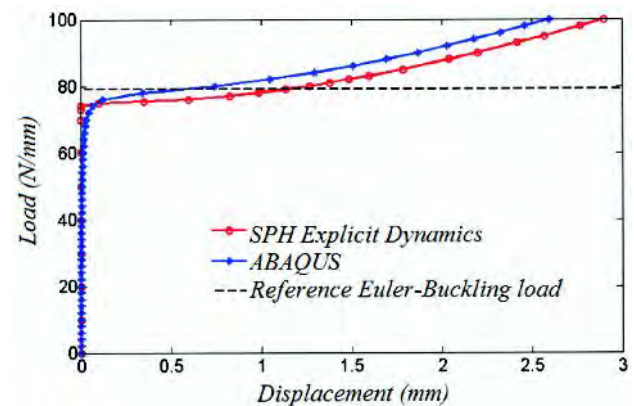


Fig. 3: Load-Deflection curve at the central plate

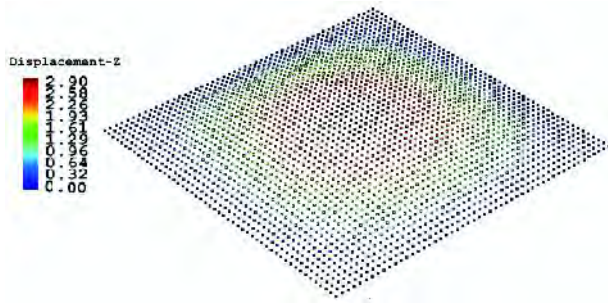


Fig. 4: Deformed configuration using the SPH shell based method ($P = 100\text{N/mm}$)

respect to the analytical solution. This critical buckling load is very close to the one obtained using ABAQUS[®] code. Figure 4 shows the final deformed configuration obtained using our in-house developed code using the SPH shell-based method, this final configuration is very close to the one obtained using FEM with ABAQUS[®] code.

4.2 Geometrically Nonlinear Analysis of a Cylindrical Roof

The second application investigates the post-buckling analysis of a hinged cylindrical roof subjected to a central pinching load as shown in Fig. 5. This benchmark has interested many researchers due to the presence of snapping behavior [14]. Owing to the tangential global stiffness matrix singularity near the limit point, the classical load controlled method is not suitable in the present case. The arc-length control method [15] is adopted for this application to achieve a complete equilibrium loading path.

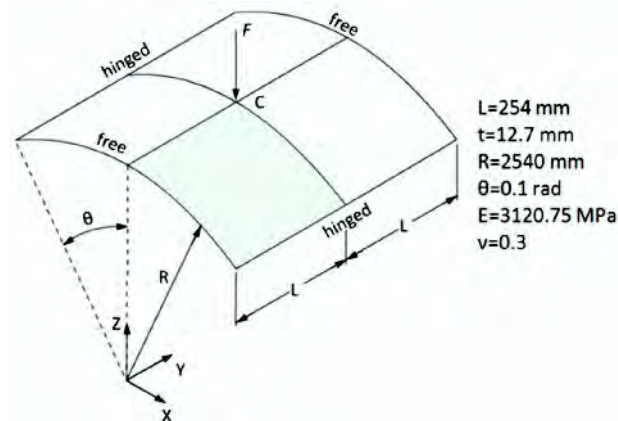


Fig. 5: Hinged cylindrical roof (geometry and material data)

Due to symmetry, only a quarter of the roof is modelled using only one layer of 21×21 particles in the SPH-shell based method. In our simulation the displacement W_C at the central point C, is controlled starting from 0 mm to 30 mm, which grows linearly until 100% in a total time of 3ms.

The load-displacement curve obtained using the SPH shell-based explicit dynamic method is given in Fig. 6 and compared to the reference solution by Klinkel [14]. As we can observe from Fig. 6, the SPH shell-based loading path solution is in a very good agreement with the reference solution. The critical displacement value obtained using SPH shell-based method is $W_C = 10.5\text{ mm}$, which is only 1.78% error compared to the reference value of Klinkel [14].

We depicted in Fig. 7a, the deformed configuration at the vicinity of the limit point obtained using the SPH shell-based method. This deformed configuration is identical to the one obtained by the FEM using S4R shell elements of ABAQUS[®] Explicit code, as indicated in Fig. 7b.

4.3 Large Deflection of Cantilever Beam Under Transverse Loads

This application deals with the geometrically nonlinear analysis of a clamped beam made of steel as shown in Fig. 8. The geometric and the material properties are as follows: beam length = 100mm, width/thickness = 10mm, Young's modulus $E = 210000\text{ Mpa}$, Poisson's ratio, $\nu = 0.3$ density $P = 7800\text{ kg/m}^3$. The beam left side is clamped while a set of transversal load increments are applied on the right free end starting from 4.375 kN to 175 kN (to satisfy the loading factor PL^2/EI ranging from 0 to 10).

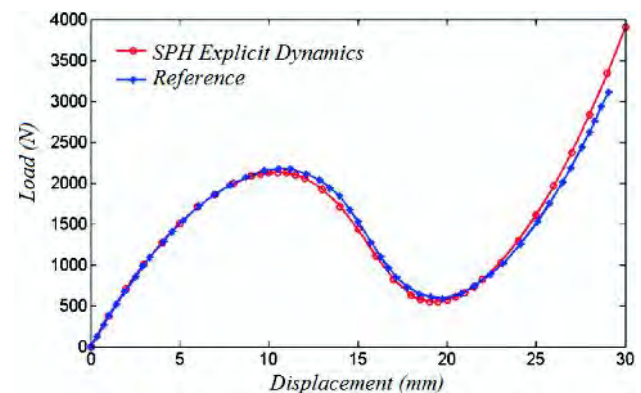


Fig. 6: Curve of Load-Displacement

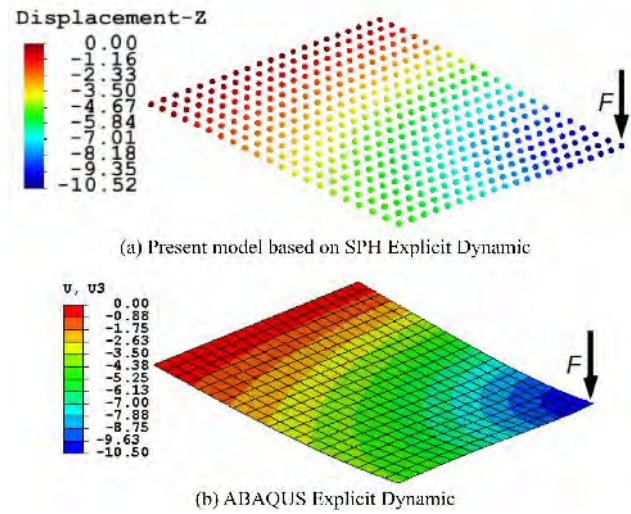


Fig. 7: Deformed configuration at the vicinity of the limit point ($W_C = 10.5$ mm)

The beam mid-surface is discretized using only one layer of 50×5 particles in the SPH-shell based method, where intermediate deformed configurations are shown in Fig. 8. The final deflection obtained using the SPH-shell based method is of 80.09 mm, which is very close to the analytical solution of Timoshenko [17] with an error of only 1.25%.

The results of beam end deflection obtained using the present SPH shell-based formulation are listed in Table 1 and compared to the analytical solution of Timoshenko [17]. As we can remark the maximal error is less than 1.25%, with a coarse discretization of only one layer of 50×5 particles. The load-deflection curve is also depicted in Fig. 9; it shows

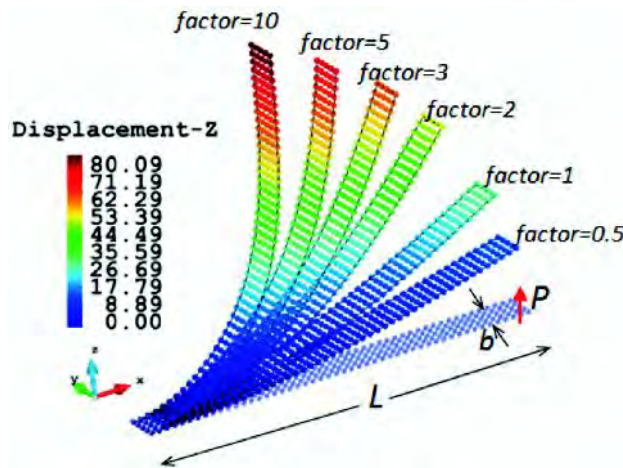


Fig. 8: Initial and deformed beam with intermediate steps

Table 1: Comparison of the SPH results with Timoshenko reference solution

P(kN)	PL^2/EI	Analytical (mm)	SPH shell (mm)	Error (%)
0	0	0	0	0.00
4.375	0.25	8.3	8.1	-2.41
8.75	0.5	16.2	15.94	-1.60
13.125	0.75	23.5	23.33	-0.72
17.5	1	30.2	30.1	-0.33
35	2	49.4	50.22	1.66
52.5	3	60.3	61.37	1.77
70	4	67	67.59	0.88
87.5	5	71.4	71.41	0.01
105	6	74.4	74.03	-0.50
122.5	7	76.7	76.01	-0.90
140	8	78.5	77.6	-1.15
157.5	9	79.9	78.94	-1.20
175	10	81.1	80.09	-1.25

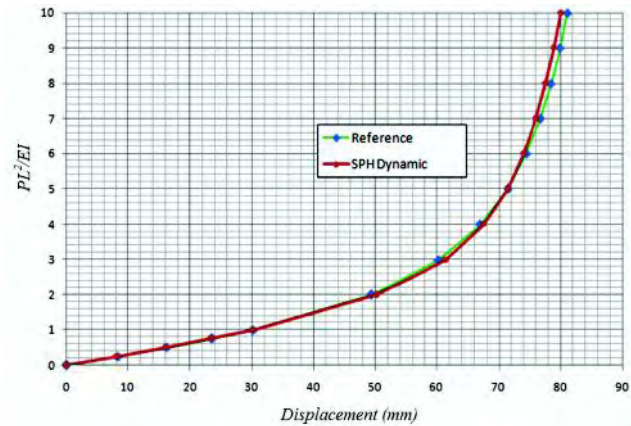


Fig. 9: Comparison of load-displacement curve (SPH vs. analytical solution)

the nonlinear character of the beam undergoing very large displacement 80% of its length. We can observe that the obtained loading path is in very good agreement with the analytical solution of Timoshenko.

4.4 Roll-up of a Clamped Plate

This benchmark represents a clamped plate under a bending moment applied at its free end. The analytical solution is given by the classical Euler formula [18], $\kappa = M/EI$, where κ is curvature and M is the bending

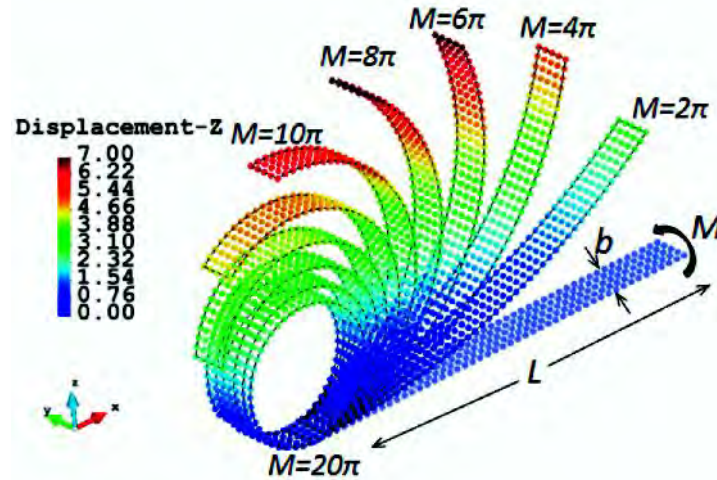


Fig. 10: Initial and intermediate configuration using the SPH shell based method

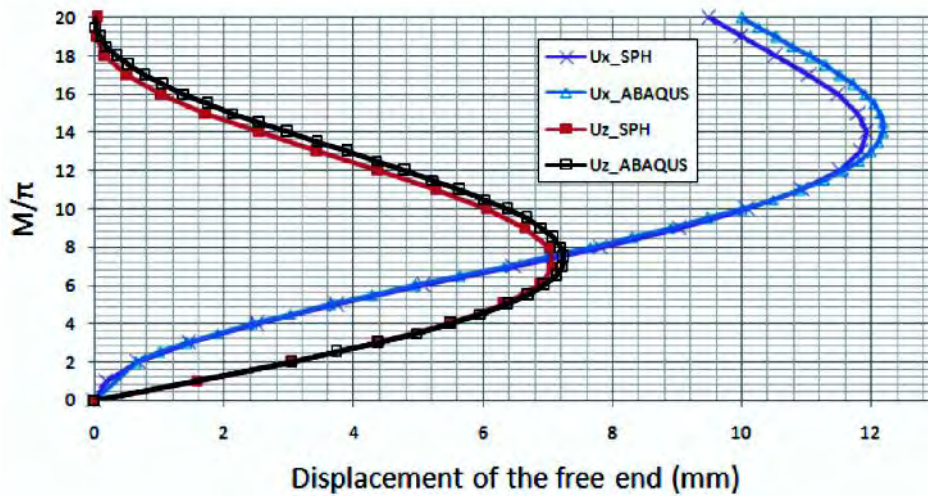


Fig. 11: Comparison of load-displacement curve (SPH vs. FEM ABAQUS)

moment. The plate geometrical and material properties are as follows: length $L = 10\text{cm}$, width/thickness $b = t = 1\text{cm}$, Young's modulus $E = 1200\text{ N/cm}^2$, Poisson's ratio $\nu = 0$. For the chosen properties, the values of the bending moment which transforms the plate into a full-circle shape is $M = 2\pi EI/L = 20\pi$.

The plate has been discretized using only one layer of 50×5 particles in the SPH-shell based method, where intermediate deformed configurations are shown in Fig. 10. For comparison purpose, the same plate has been modelled using ABAQUS® code with a FE mesh of 50×5 S4R shell elements.

The results of beam free end vertical and horizontal displacements obtained using the present

SPH shell-based formulation are depicted in Fig. 11 and compared to the FE solution of ABAQUS. As we can observe, the loading path obtained using our SPH method shows the very nonlinear behaviour of the solution due to the very large rotation up to 2π . The predicted solution using the developed SPH shell-based formulation is very close to the FE loading path predicted using ABAQUS code, even if we use a coarse discretization of one layer of 50×5 particles. This demonstrates the effectiveness and the efficiency of the present method compared to the FEM and we can clearly say that the proposed SPH shell based method can represent a promising alternative to the classical FEM.

5. Concluding Remarks

In this investigation, we present a new modified SPH shell-based formulation for the geometrically nonlinear analysis of the thin-walled structures, especially useful for the post-buckling analysis of shell-like structures. As explained above, the two major drawbacks of the classic SPH method were eliminated using the corrected kernel functions and by adopting the Total Lagrangian Formulation. Based on the Mindlin-Reissner shell theory, the Continuum SPH method has been modified to reproduce the shell behaviour, by modelling a thin shell structure with only one layer of particles located into the shell mid-surface. Moreover, the Rodrigues formula associated with the incremental large rotations was introduced to update the pseudo-normal vector; this allows the method to

be very efficient in dealing with structures undergoing very large three dimensional rotations up to 2π .

To validate the proposed SPH shell-based method, several numerical applications have been treated using the explicit dynamic time integration scheme. The obtained results were compared to the analytic solutions as well as to numerical reference solution of the FEM using ABAQUS code or from the literature. It has been shown, through the numerical applications that the SPH shell-based method using only one layer of particles is very suitable for the study of shell-like structures and presents a very strong potential especially in terms of CPU time saving while keeping a high level of accuracy when compared to the classical SPH method and to the classical Finite Element method.

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