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Research Paper

Plasticity: A Journey from Uniaxial Stress to Uniaxial Strain

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The design of conventional civil structures and machines is generally based on uniaxial stress experiments in which strength of the material is determined with the help of a standard tensile test on a Universal Testing Machine. This strength value and corresponding constitutive relation is applicable only for the structures which are subjected to very low strain rates (10^{-6} to 10^{-2} s^{-1}). The behavior of solid materials differs considerably under dynamic loading conditions viz. ballistic impact of a bullet or blast loading. The constitutive model for the material needs to be updated to include the effect of large plastic deformations, high strain rates and temperature. One of the popular experimental techniques to study material behaviour under high strain rates is Split Hopkinson Pressure Bar, which is also based on uniaxial stress assumption in the specimen.

But the behaviour of material under hypervelocity impacts demands an entirely different approach as the impact pressures generated are orders of magnitude higher than the static strength of material. Shock waves are formed in the solid and the material starts behaving like a fluid. The volumetric component of stress i.e. hydrostatic pressure can no longer be neglected and an equation of state (EOS) is now required to mathematically model the hydrodynamic behaviour of material in addition to constitutive and failure model. This situation demands a shift from conventional uniaxial stress experiments in plasticity to an entirely new domain of uniaxial strain experiments.

Key Words : Plasticity; Uniaxial Stress; Ballistic Impact; Hyper-Velocity Impact; Shock Waves; Uniaxial Strain

1. Introduction

Most of the plasticity is based on experimental results which are generated from uniaxial stress experiments on small sized material specimens. One of the fundamental experiments in the study of plastic behaviour of material is a tensile test which dates back to Leonardo da Vinci (1452-1519). He was probably the first researcher to study the strength of a material. Fig. 1 is taken from his notebooks [1] and it shows the experimental set-up used by Vinci to determine the strength of iron wires. A basket is suspended from the steel wire whose strength is to be determined. The basket is slowly filled with sand until the wire breaks and the weight of the sand gives the strength of the wire.

The next giant leap in the study of material behaviour was experiments conducted by Galileo Galilei (1564-1642). He conducted systematic experiments on tensile strengths of bars and bending strengths of beams. The experimental set-ups used by Galileo for tensile test and bending test are shown in Fig. 2 and Fig. 3 respectively. He concluded that the strength of bar under tensile load is independent of its length and depends on its cross-sectional area. He also concluded that geometrically similar beams are not equally strong. These experiments are discussed in his book "Dialogues Concerning Two New Sciences" [2].

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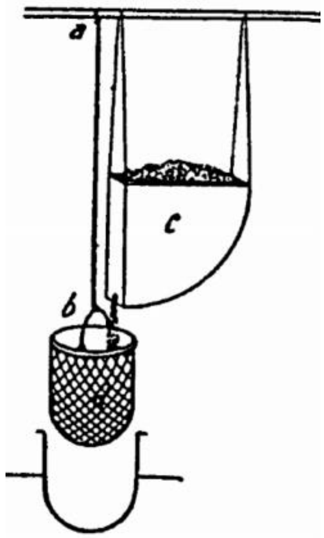

Fig. 1: The schematic of tensile test on steel wires by Leonardo da Vinci

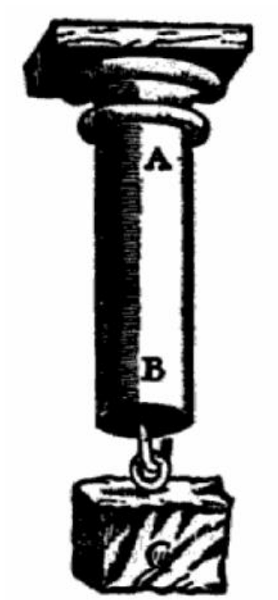

Fig. 2: The schematic of tensile test on bars by Galileo Galilei

Modern tensile tests are carried out using Universal Testing Machines (UTM). These machines provide the complete stress-strain diagram for a material which can be used to derive various mechanical properties of that material. Robert Hooke (1635-1703) gave the mathematical relation between the applied force and elastic deformation of a material and this linear relationship is known as Hooke's law which is the basic constitutive relation between stress

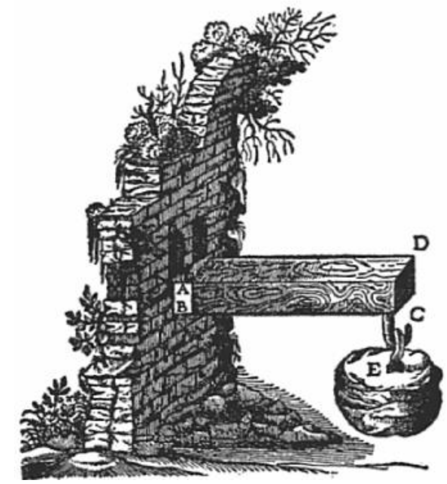

Fig. 3: The schematic of bending test on bars by Galileo Galilei

and strain in a material within elastic limits.

The strain hardening effect in the plastic behaviour of an isotropic material is given by Ludwik-Hollomon equation: $\sigma = \sigma_0 + K\varepsilon^n$, where σ_0 is the yield stress, 'K' is the strain hardening coefficient and 'n' is the strain hardening exponent.

All the experiments discussed until now are uniaxial stress experiments i.e. only stress acting on the material is longitudinal stress (σ_x) and all other stress components are assumed to be zero. This can be achieved by applying longitudinal force on bar geometries where length of the bar is significantly greater than the diameter.

The constitutive relations like Hooke's law and Ludwik-Hollomon equation are applicable only for those structures where the loading is quasi-static in nature i.e. strain rate is very low (10^{-6} to 10^{-3} s^{-1}). But in defence applications like ballistic impact on an armour material, strain rates are much higher (10^3 to 10^4 s^{-1}). The constitutive model for the material needs to be updated to include the effect of large plastic deformations, high strain rates and temperature.

The situation becomes more complicated in the case of hypervelocity impact where impact pressure generated are so high that there is shock waves propagation in the material and the material starts behaving like a fluid.

The experimental techniques and constitutive equations describing the material behaviour under ballistic impact and hypervelocity impact are discussed in the following sections.

2. Material Behaviour Under Ballistic Impact

Ballistic impact can be defined as the impact of a projectile moving within the velocity range of 200 to 2000m/s. These velocities are called ordnance velocities since this is the usual velocity range of conventional firearms.

For example, in Indian context, 0.38 ball ammunition from a revolver has a velocity of 183±9m/s whereas 5.56 x 45 Ball ammunition of INSAS rifle has a velocity of 890±10m/s. Main Battle Tank (MBT) ARJUN developed by Defence Research and Development Organization (DRDO) uses a Fin Stabilized Armour Piercing Discarding Sabot (FSAPDS) projectile which has a muzzle velocity of 1650m/s.

It is a well known fact that the material behaviour changes with the rate of loading. Fig. 4 shows the schematic of material behaviour under low velocity impact and ballistic impact. Strain rates in ballistic impact phenomena usually lie in the range from 10^3 to 10^4 s^{-1}, whereas the tensile tests in Universal Testing Machines (UTM) are carried out in the strain rate range from 10^{-4} to 10^{-2} s^{-1}. Hence, it is very dangerous to use the material data from UTM tests to design armour systems which are supposed to be subjected to ballistic impacts.

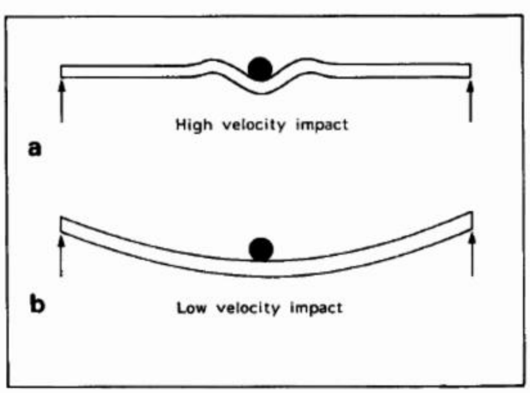

Fig. 4: Material behaviour under low velocity impact and ballistic impact

2.1 Constitutive & Damage Modelling for Ballistic Impact

In the last few decades, the availability of computational resources has increased exponentially. A number of commercially available finite element programs (ABAQUS, ANSYS, LS-DYNA etc.) have been developed to simulate the response of various materials under static as well as dynamic loading conditions.

But the accuracy of the output from these programs depends on the accuracy of input regarding material description. The material description can be provided in terms of a constitutive model and a damage model. Constitutive model provides relationship between stress, strain, strain rate and temperature whereas damage model provides information about the fracture behaviour of the material.

A number of constitutive and damage models have been developed in the past for different materials like metals, polymers, composites and ceramics. The discussion in this paper will be restricted for metals only.

One of the most commonly used constitutive and damage model for metals has been proposed by G R Johnson and W H Cook [3, 4]. The constitutive model given by Johnson and Cook is:

$$\sigma = [A + B\varepsilon^n][1 + C\ln\dot{\varepsilon}^*][1 - T^{*m}] \qquad (2.1)$$

where, σ is the stress, ε is effective plastic strain, and $\dot{\varepsilon}^*$ is the ration of strain rate to the reference strain rate and T^{*m} is the homologous temperature. There are 05 material parameters *A, B, C, n* and *m* which are to be determined experimentally. A is yield stress, *B* is the strain hardening coefficient, *n* is the strain hardening exponent, *C* is the strain rate sensitivity coefficient and *m* is the thermal softening coefficient.

On the similar lines, Johnson and Cook also proposed a fracture model based on the dependence of fracture strain on stress triaxiality, strain rate and temperature.

$$\varepsilon^f = [D_1 + D_2 e^{D_3\sigma^*}][1 + D_4\ln\dot{\varepsilon}^*][1 + D_5 T^*] \qquad (2.2)$$

where, ε^f is the fracture strain and σ^* is the stress triaxiality ratio. Once again, there are 05 material parameters D_1, D_2, D_3, D_4 and D_5 which are to be determined experimentally.

2.2 Experimental Techniques for High Strain Rate Studies

Standard Universal Testing Machines (screw driven) can be used to carry out tensile tests from strain rates of 10^{-6} to 10^{-2} s^{-1}. High speed Universal Testing Machines (servo-hydraulic) can go upto strain rates of 1 s^{-1}. The experimental data from these machines can be used for quasi-static loading conditions or low velocity impacts. But strain rates in ballistic impact conditions lies between 10^3 to 10^4 s^{-1}. One of the most common techniques for tensile and compressive testing at high strain rates is Split Hopkinson Pressure Bar, which is available in our lab.

2.3 Split Hopkinson Pressure Bar (SHPB)

Hopkinson Bar is named after B Hopkinson [5] who, in 1914, developed pressure bar technique to experimentally determine the pressure produced by an explosive. In 1949, Kolsky [6] used the *pressure bar* technique in determining the dynamic compression stress-strain behaviour of different materials by splitting the Hopkinson bar in two parts. A schematic diagram of classical SHPB is shown in Fig. 5. A detailed discussion on the history and recent developments has been given by Gama *et al.* in their review paper [7].

The compression Hopkinson bar test apparatus at TBRL Chandigarh consists of two elastic pressure bars between which the rectangular specimen is sandwiched. The bars used are constructed from high strength maraging steel having yield strength ~ 1750 MPa. The yield strength of the bar material determines the maximum stress attainable within the deforming specimen given that the bars remain within elastic limit. The diameter of striker, incident and transmitter bars is 20 mm while length of the incident and transmitter bars is 2000 mm each.

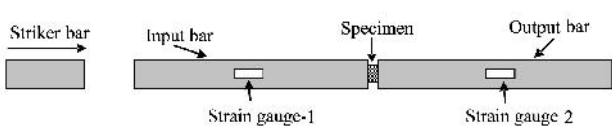

Fig. 5: Schematic of split Hopkinson pressure bar

A projectile or striker bar of same diameter as the input bar is propelled towards the incident bar using compressed gas launcher. The length of the striker bar can be varied from 100mm to 900mm. Upon impact a longitudinal elastic compressive wave is generated within the incident bar designated as $\varepsilon_i(t)$. At the bar-specimen interface, a part of the pulse, designated as $\varepsilon_r(t)$ is reflected back in the incident bar and the remaining of the stress pulse passes through the specimen and upon reaching the transmitter bar is termed as transmitted wave, $\varepsilon_t(t)$.

The stress and strain in the specimen are calculated from the transmitted and reflected wave signals respectively as follows:

Stress in the specimen, $$\sigma = \frac{AE\varepsilon_t}{A_s} \tag{2.3}$$

Stress in the specimen, $$\varepsilon = \frac{2C_0}{l_s}\int \varepsilon_r dt \tag{2.4}$$

where ε_i is the transmitted wave, ε_r is the reflected wave, E is the Young's Modulus of bar material, C_0 is the sound velocity in bar material, l_s is the length of specimen and A & A_s are area of cross-section of bar & specimen respectively.

The experimental data generated from low strain rate tests using screw driven UTM, intermediate strain rate tests using servo-hydraulic UTM and high strain rate tests using Split Hopkinson Pressure Bar can be combined to develop constitutive and damage models for simulating the behaviour of materials under ballistic impact.

The one thing that is common in all the experiments discussed above is that they are carried out under uniaxial stress conditions. But hypervelocity impacts require an entirely different approach in order to study the material behaviour.

3. Material Behaviour Under Hypervelocity Impact

Impacts with velocity greater than 3 km/s are usually treated as hypervelocity impacts [8]. Natural examples of hypervelocity impacts can be impacts of space debris on a satellite or a spacecraft or impact of a meteorite on earth. In context of defence applications, Inter-Continental Ballistic Missiles (ICBM) can travel at the velocity of 4 to 8 km/s. The relative velocity of impact in case of an interceptor missile hitting an enemy missile can go upto 10 km/s.

The impact pressure generated during hypervelocity impact is so high that it exceeds the material strength by order of magnitudes. Shock waves are produced inside the projectile as well as target materials and solid materials behave like fluids at such high pressures and temperatures. The response of the solid material to ballistic impact and hypervelocity impact is shown in Fig. 6 and Fig. 7 respectively.

Fig. 6 shows the ballistic impact of AK-47 bullet (7.62 x 39mm) on an aluminium plate with a velocity is 710±10 m/s. The experiment was carried out at High Strain Rate Studies Division in TBRL Chandigarh. The crater diameter created by the bullet is almost equal to the bullet diameter (7.62mm).

Fig. 6: Ballistic Impact of AK-47 bullet on an aluminium target

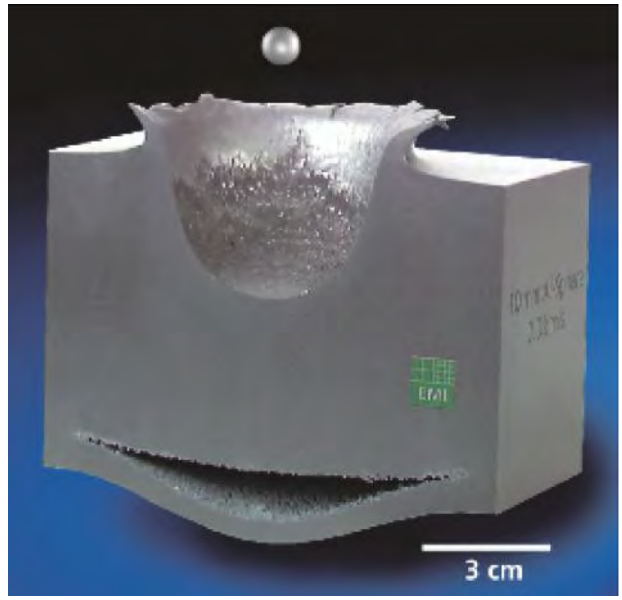

Fig. 7: Hypervelocity impact of 10mm aluminium sphere on an aluminium target

Fig. 7 shows the hypervelocity impact of a 10mm aluminium sphere on an aluminium plate with a velocity of 7km/s [9]. The crater diameter in this case is around 70mm, which is 7 times more than the projectile diameter. This shows the hydrodynamic behaviour of material which causes material to flow in radial direction under hypervelocity impact.

Now since the solid material is not behaving as a solid, the principles of solids mechanics are not applicable any more under such circumstances. The problem can be analyzed using hydrodynamic behaviour by ignoring material strength completely.

But if the target is thick enough, the projectile velocity will reduce during its course of penetration and may come in the ballistic impact regime or may be further reduced to zero if the projectile is not able to perforate the target.

Hence, for analyzing hypervelocity impacts, hydrodynamic behaviour as well as material strength needs to be considered. The material strength is represented by its constitutive model and damage model. The hydrodynamic behaviour is represented by an equation of state which relates the hydrostatic stress (pressure) to density and temperature.

3.1 Equation of State for Solids

No generalized equation of state is available for solids as compared to gases, where ideal gas equation (PV= nRT) can be applied to all the gases. One of the extensively used equation of state for solids under hypervelocity impacts is Mie-Gruneisen Equation of state [10]. The general form of the Mie-Gruneisen equation of state is:

$$p - p_{ref} = \Gamma \rho (e - e_{ref}) \tag{3.1}$$

where, p is the pressure, p_{ref} is the reference pressure, e is the specific internal energy, e_{ref} is the reference specific internal energy, ρ is the density and Γ is called Gruneisen parameter. The reference values for pressure and specific internal energy are taken on Shock Hugoniot curve of that material.

3.2 Shock Hugoniot

During hypervelocity impact, shock waves are produced in projectile as well as target materials. Shock waves are nothing a moving sharp vertical front across which but a discontinuity in pressure, temperature and density of the material. Fig. 8 shows a shock front moving with shock velocity U_s through a compressible material. The conservation of mass, momentum and energy across the shock front results in three conservation equations which are known as Rankine-Hugoniot jump conditions [11].

$$\rho_0 U_s = \rho_1 (U_s - U_p) \tag{3.2}$$

$$p_1 - p_0 = \rho_0 U_s U_p \tag{3.3}$$

$$p_1 U_p = \frac{1}{2} \rho_0 U_s U_p^2 + \rho_0 U_s (e_1 - e_0) \tag{3.4}$$

where, p_0, p_1, e_0, e_1, ρ_0, and ρ_1 are pressure, internal energy and density ahead and behind the shock front respectively.

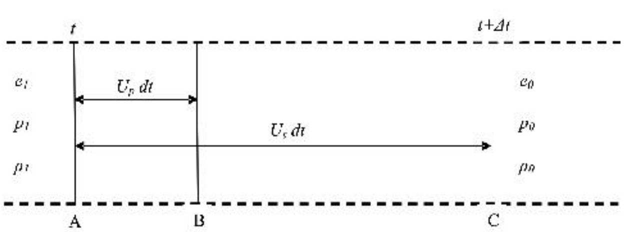

Fig. 8: Schematic of a shock front propagating through a material from A to C

If the initial state of material ahead of shock wave is known, then the above 3 equations (3.2, 3.3 and 3.4) have 5 unknown variables p_1, e_1, ρ_1, U_s and U_p. So, one more equation is required in addition to boundary conditions so as to determine all the variables. That equation is determined experimentally and is often called shock Hugoniot.

Shock Hugoniot can be a relation between any 2 variables from p_1, ρ_1, U_s and U_p. But it is convenient to determine U_s–U_p relation experimentally. The relation between U_s and U_p is found to be linear for most of the materials. Mathematically,

$$U_s = C_0 + SU_p \tag{3.5}$$

3.2 Equation of State in terms of Shock Hugoniot

Equation (3.3) can be re-written in the following form:

$$U_p = \eta U_s \tag{3.6}$$

where,

Substituting equation (3.6) in equation (3.5), we get

$$U_s = \frac{C_0}{1 - s\eta} \tag{3.7}$$

Substituting equation (3.6) and (3.7) in equation (3.3), we get

$$p_h = p_1 - p_0 = \frac{\rho_0 c_0^2 \eta}{(1 - S\eta)^2} \tag{3.8}$$

Eliminating U_s and U_p from equation (3.4), we can write

$$e_h = e_1 - e_0 = \frac{p_h \eta}{2\rho_0} \tag{3.9}$$

Substituting equation (3.8) and (3.9) in equation (3.1), we can write Mie-Gruneisen EOS as:

$$p = p_h + \Gamma \rho (e - e_h)$$

$$p = \frac{\rho_0 C_0^2 \eta}{(1 - s\eta)^2}\left(1 - \frac{\Gamma_0 \eta}{2}\right) + \Gamma_0 \rho_0 e$$

So, equation of state for any isotropic material can be found out if Shock Hugoniot for that material (Eq 3.5) is determined experimentally.

3.2 Experimental Determination of Shock Hugoniot

Shock Hugoniot can be a relation between any 2 variables from p_1, ρ_1, U_s and U_p. But it is convenient to determine U_s–U_p relation experimentally. It is not possible to generate shock waves in uniaxial stress experiments because shock waves are generated at pressures which are orders of magnitude higher than the strength of material.

Fig. 9 shows the typical stress-strain curve for an isotropic material under uniaxial stress conditions. As we increase the stress on the material from σ_0 to σ_2, the slope of the curve and hence the velocity of the stress wave decreases ($V_2<V_1<V_0$). This results in a dispersive wave structure and no shock wave front is formed in uniaxial stress conditions [12].

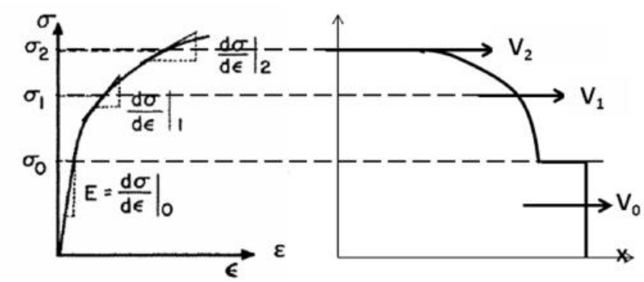

Fig. 9: Stress-Strain curve for a material under uniaxial stress conditions

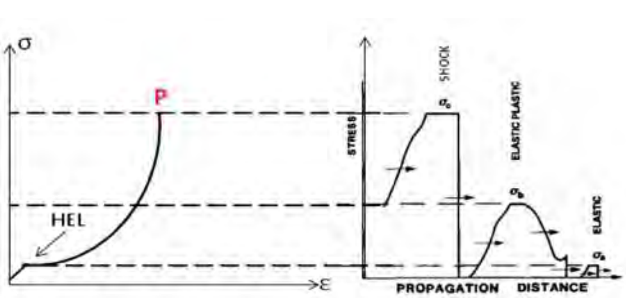

Fig. 10: Stress-Strain curve for a material under uniaxial strain conditions

Fig. 10 shows the typical stress-strain curve for an isotropic material under uniaxial strain conditions [13]. The point where material changes its behaviour from elastic to plastic is called Hugoniot Elastic Limit (HEL). But the plastic region of the curve in uniaxial strain conditions is concave upwards as compared to convex upwards under uniaxial stress conditions. This results in increase in the wave velocity as the stress is increased. The faster moving plastic waves at higher stress levels catches up with the slower moving waves at lower stress levels which ultimately results in a sharp wave front called shock front.

The experimental set-up for a uniaxial strain experiment is shown in Fig. 11. A flyer plate is impacted on a target plate, which is instrumented with shock arrival pins to measure the shock wave velocity and flyer velocity. The flyer may be of the same material as the target plate and the impact in that case is called a symmetric impact. Symmetric impact provides convenience in determining the particle velocity, which comes out to be half of the flyer velocity.

Two techniques are used to accelerate the flyer plate to its desired velocity: gas gun or explosive. Gas Guns provide better control on the velocity and angle of impact whereas explosive technique can provide higher velocities than gas guns (upto 8km/s). Both kind of techniques are used in TBRL to carry out shock wave studies on various materials like metals and ceramics.

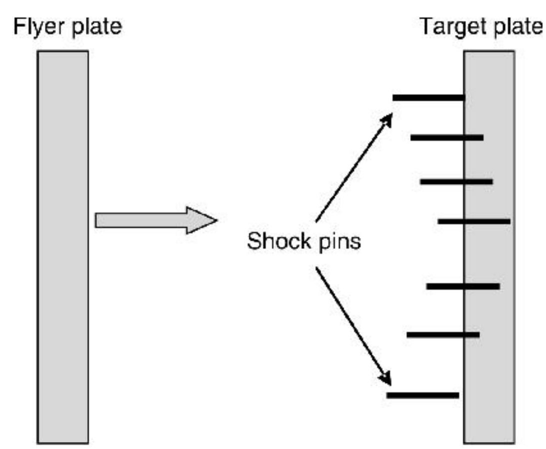

Fig. 11: Schematic of plate impact experiment

Uniaxial strain experiments are much more difficult to perform than uniaxial stress experiments because the time scales available for measurement are in micro-seconds. Moreover, one plate impact experiment provides one point on the U_s–U_p shock Hugoniot whereas a uniaxial stress experiment provides complete stress-strain curve in one shot. A minimum of 4-5 experiments are required to generate shock Hugoniot for any material.

3. Conclusions

Material behaviour under quasi-static loading, low velocity impact and ballistic impact can be characterized by carrying out uniaxial stress experiments using universal testing machines and Split Hopkinson Pressure Bars. But hypervelocity impact behaviour demands a shift from traditional uniaxial stress experiments to an entirely different domain of uniaxial strain experiments where applied pressures can exceed the material strength by orders of magnitude. The material description under hypervelocity impact requires an equation of state in addition to constitutive relations and failure models. There is no universal equation of state available for solids as compared to gases where an ideal gas equation (PV = nRT) is available. Mie-Gruneisen equation of state is the most popular equation of state which is applicable for inert materials with no phase changes involved. The reference state in this equation of state is taken on Hugoniot curve which is generated using plate impact experiments.

Uniaxial stress experiments have been traditionally the domain of mechanical and civil engineers whereas uniaxial strain experiments have been the domain of shock physicists. These domains need to be integrated in order to design armour systems which can provide protection against hypervelocity impacts.

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