EULER'S TOTIENT FUNCTION AND ITS INVERSE

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Euler's totient function \( \phi(n) \) can be defined for all positive integral values of \( n \) by the relations:

(1) \( \phi(1) = 1 \):

and for any prime \( p \) and \( u \geq 1 \)

(2) \( \phi(pu) = p\phi(u) \) or \( (p - 1) \phi(u) \)

according as \( p \) does or does not divide \( u \).

The simple reduction formula (2) which is so suitable for computing the values of \( \phi(n) \), does not appear to have been stated before.

A method of finding all the elements of the set

\( \phi^{-1}(m) = \{n : \phi(n) = m\} \)

is described, and the conjecture that the inequality

\( \phi(x) < m \)

has about twice as many even solutions than it has odd ones is shown to be true. It is also shown that sets \( \phi^{-1}(m) \) with all elements even exist.

1. INTRODUCTION

In what follows, small letters denote positive integers; \( p \)'s denote primes; \( E \) denotes an empty set; and if \( A \) is any set, then

\[ tA = \{ta : a \in A\}. \tag{1.1} \]

For any given positive integer \( n \), Euler's totient function \( \phi(n) \) denotes the number of positive integers which are prime to \( n \) and do not exceed \( n \).

The following properties of \( \phi(n) \) are well-known and are stated here for ready reference:

(i) \( \phi(1) = 1 = \phi(2) \). \tag{1.2}

(ii) \( \phi(n) \) is multiplicative, i.e. for \( (n_1, n_2) = 1 \),

\[ \phi(n_1n_2) = \phi(n_1) \phi(n_2). \tag{1.3} \]

(iii) For any prime \( p \),

\[ \phi(p^s) = p^{s-1}(p - 1). \tag{1.4} \]
From (1.3) and (1.4), it follows that
\[
\phi(n) = n \prod_{p \mid n} (1 - p^{-1}).
\] ...(1.5)

(iv) \(\phi(n)\) is even for each \(n \geq 3\).

This means that there is no \(x\) for which
\[
\phi(x) = 2t + 1, \ t \geq 1.
\]

On the other hand,
\[
\phi(x) = 2t, \ t \geq 1
\]
may or may not have any solution for a given \(t\) and if it has a solution, it may not be unique.

Thus \(\phi(x) = 6\) has exactly four solutions viz. \(x = 7, 9, 14, 18\); while there is no \(x\) for which \(\phi(x) = 14\).

(v) If for some \(n\), \(\phi(n) = m\), then
\[
\phi(2n) = m \text{ if and only if } n \text{ is odd.}
\] ...(1.7)

2. A Reduction Formula for \(\phi(n)\)

We have \(\phi(1) = 1\):

for \(n \geq 2\), we can write
\[
n = pu
\]
where \(p\) is a prime divisor of \(n\) and \(u\) is some integer \(\geq 1\). Then it is easy to see that
\[
\phi(n) = p\phi(u) \text{ or } (p - 1) \phi(u)
\] ...(2.1)
according as \(p\) does or does not divide \(u\).

(In practice it is best to take \(p\) as the smallest prime divisor of \(n\).)

With \(\phi(1) = 1\), (2.1) completely defines \(\phi(n)\) for all positive integral values of \(n\). Actually, (2.1) provides a simple reduction formula for \(\phi(n)\). This formula does not appear to have been given before.

After the author had computed values of \(\phi(n)\) for \(n \leq 7500\) manually, Ajeet Singh of the Moti Lal Engineering College, Allahabad and Nirmal Roberts of the Computer Centre at the I.I.T., Kanpur, were able to write independently programmes for the computation of \(\phi(n)\). These were based directly on (2.1). Finally, Nirmal produced a table of values of \(\phi(n)\) for \(n \leq 25000\) which is a considerable extension of available tables.

3. The Set \(\phi^{-1}(m)\)

For any given \(m\), we define \(\phi^{-1}(m)\) by the relation:
\( \phi^{-1}(m) = \{ n : \phi(n) = m \} \)  \hspace{1cm} \ldots(3.1)

This set is empty for all odd values of \( m > 1 \) and for many even values of \( m \) also. Our interest will be mainly in those values of \( m \) for which the set is non-empty.

**Theorem 1** — Any non-empty set \( \phi^{-1}(m) \) is bounded both above and below.

**Proof:** Let \( n \) be any element of \( \phi^{-1}(m) \).

Evidently, then \( m \leq n \).

The set is therefore, bounded below.

Again, from (1.5), we have

\[
n/\phi(n) = \Pi_{p | n} p/(p - 1) \leq \Pi_{(p-1) | m} p/(p - 1). \hspace{1cm} \ldots(3.3)
\]

This follows from the fact that if \( p | n \), then \( (p - 1) | m \); but if \( (p - 1) | m \), then \( p \) may or may not divide \( n \).

Hence, no \( n \) for which \( \phi(n) = m \), can exceed \( U(m) \) where

\[
U(m) = m \Pi_{(p-1) | m} p/(p - 1). \hspace{1cm} \ldots(3.4)
\]

This completes the proof of the theorem.

**Corollary** — If \( q \) is the largest odd element of \( \phi^{-1}(m) \), then

\[
q \leq U(m)/2. \hspace{1cm} \ldots(3.5)
\]

(This follows from the fact that \( 2q \) is also an element of \( \phi^{-1}(m) \).)

We can find another upper bound for \( n/\phi(n) \) as follows. Let \( P_k \) denote the product of the first \( k \) primes

\[
p_1^*, p_2^*, p_3^*, \ldots, p_k^*
\]

with \( p_1^* = 2, p_2^* = 3, p_3^* = 5 \), and so on.

Then for any \( n \) for which \( P_k \leq n < P_{k+1} \), we have

\[
n/\phi(n) \leq P_k/(p_1^* - 1) (p_2^* - 1) \ldots (p_k^* - 1). \hspace{1cm} \ldots(3.6)
\]

This follows from the fact that no \( n \) in the said interval can have more than \( k \) distinct prime divisors.

The sign of equality actually holds in (3.6) for \( n = P_k \) and may be for several other numbers too.
Results (3.4) and (3.6) can often be used together with advantage. Take $m = 192$ for example. Then using (3.4), we find that no element of $\phi^{-1}(m)$ can exceed

$$192 \cdot (2/1) \ (3/2) \ (5/4) \ (7/6) \ (13/12) \ (17/16) \ (97/96) \ (193/192)$$

which is just less than 983.

Since 983 lies between $P_4$ and $P_5$ and for any $n$ in this interval

$$n / \phi(n) \leq (2/1) \ (3/2) \ (5/4) \ (7/6),$$

no element of $\phi^{-1}(192)$ can exceed 840.

Our tables show that 840 is actually the largest element of $\phi^{-1}(192)$ (this is, however, purely a matter of chance and we cannot assert that this will always be so). For $m = 400$, (3.4) gives 1820 as an upper bound for the set $\phi^{-1}(400)$. Use of (3.6) improves it to 1750 while the largest element of $\phi^{-1}(400)$ is 1650.

4. Determination of $\phi^{-1}(m)$

Let $n$ be an element of $\phi^{-1}(m)$ for a given $m$. Assume that $p$ is the least prime divisor of $n$. Let

$$n = p^d u, \text{ where } (u, p) = 1.$$ 

This clearly implies that $u$ has no prime divisor $\leq p$.

Evidently, we have

$$m = \phi(n) = \phi(p^d) \phi(u). \quad \text{ ...(4.1)}$$

For (4.1) to hold, it is necessary that our $p$ be such that

$$(p - 1) \mid m \quad \text{ ...(4.2)}$$

and $u$ belong to that subset of $\phi^{-1}(m/\phi(p^d))$ which consists of those of its elements which have no prime divisor $\leq p$. Such a subset can conveniently be denoted by $\phi_p^{-1}(m/\phi(p^d))$. It will be clear that every element of

$$p^d \phi^{-1}(m/\phi(p^d)) \quad \text{ ...(4.3)}$$

gives a solution of the equation

$$\phi(x) = m. \quad \text{ ...(4.4)}$$

In fact, (4.3) provides all those solutions of (4.4) which have $p$ as their least prime divisor and $p^d$ as the highest power of $p$ which divides them.

Letting $p$ run through all those primes which satisfy condition (4.2) and $d$ through all those values for which $\phi(p^d)$ divides $m$, all the solutions of (4.4) can be
obtained. These determine $\phi^{-1}(m)$. For any prime $p$ satisfying (4.2), we can ignore all those values of $d$ for which $m/\phi(p^d)$ is an odd number $> 1$.

For reasons which will be clear a little later, it will be best to consider values of $p$ in descending order of magnitude and those of $d$ in an ascending order.

The following example will clarify the procedure.

*Example* — Take $m = 576$.

To get the primes $p$ for which $(p - 1) | m$, we write out all the divisors of $m$; add 1 to each one of them and retain the primes alone. Now, $576 = 2^6 \cdot 3^2$, the divisors of 576, therefore are:

$$1, 2, 4, 8, 16, 32, 64; 3, 6, 12, 24, 48, 96, 192; 9, 18, 36, 72, 144, 288, 576.$$

Adding 1 to each of these, we get

$$2, 3, 5, 9, 17, 33, 65; 4, 7, 13, 25, 49, 97, 193; 10, 19, 37, 73, 145, 289, 577.$$

The primes in this list arranged in descending order are:

$$577, 193, 97, 73, 37, 19, 17, 13, 7, 5, 3, 2.$$

We assume that sets $\phi^{-1}(x)$ are available for all $x < 576$. Those that we shall need are:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\phi^{-1}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{1, 2}</td>
</tr>
<tr>
<td>6</td>
<td>{7, 9, 14, 18}</td>
</tr>
<tr>
<td>8</td>
<td>{15, 16, 20, 24, 30}</td>
</tr>
<tr>
<td>16</td>
<td>{17, 32, 34, 40, 48, 60}</td>
</tr>
<tr>
<td>18</td>
<td>{19, 27, 38, 54}</td>
</tr>
<tr>
<td>32</td>
<td>{51, 64, 68, 80, 96, 102, 120}</td>
</tr>
<tr>
<td>36</td>
<td>{37, 57, 63, 74, 76, 108, 114, 126}</td>
</tr>
<tr>
<td>48</td>
<td>{65, 104, 105, 112, 130, 140, 144, 156, 168, 180, 210}</td>
</tr>
<tr>
<td>72</td>
<td>{73, 91, 95, 111, 117, 135, 146, 148, 152, 182, 190, 216, 222, 228, 234, 252, 270}</td>
</tr>
<tr>
<td>96</td>
<td>{97, 119, 153, 194, 195, 208, 224, 238, 260, 280, 288, 306, 312, 336, 360, 390, 420}</td>
</tr>
<tr>
<td>144</td>
<td>{185, 219, 273, 285, 292, 296, 304, 315, 364, 370, 380, 432, 438, 444, 456, 468, 504, 540, 546, 570, 630}</td>
</tr>
<tr>
<td>288</td>
<td>{323, 365, 455, 459, 555, 584, 585, 592, 608, 646, 728, 730, 740, 760, 864, 876, 888, 910, 912, 918, 936, 1008, 1080, 1092, 1110, 1140, 1170, 1260}</td>
</tr>
</tbody>
</table>
Our calculations can now be presented in the following tabular form:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$d$</th>
<th>$m/\phi(p^d)$</th>
<th>$p^d\phi^{-1}(m/\phi(p^d))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>577</td>
<td>1</td>
<td>1</td>
<td>577 ${1} = {577}$</td>
</tr>
<tr>
<td>193</td>
<td>1</td>
<td>3</td>
<td>Discarded</td>
</tr>
<tr>
<td>97</td>
<td>1</td>
<td>6</td>
<td>97. $E$</td>
</tr>
<tr>
<td>73</td>
<td>1</td>
<td>8</td>
<td>73. $E$</td>
</tr>
<tr>
<td>37</td>
<td>1</td>
<td>16</td>
<td>37. $E$</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>32</td>
<td>19. $E$</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>36</td>
<td>17 ${37} = {629}$</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>48</td>
<td>13. $E$</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>96</td>
<td>7 ${97} = {679}$</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>144</td>
<td>5. $E$</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>288</td>
<td>3 ${323, 365, 455} = {969, 1095, 1365}$</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>96</td>
<td>9 ${97, 119} = {873, 1071}$</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>32</td>
<td>27. $E$</td>
</tr>
</tbody>
</table>

At the next step, we need all the odd elements of $\phi^{-1}(576)$ and these have already become available. This explains why we decided to consider the primes in descending order.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$d$</th>
<th>$m/\phi(p^d)$</th>
<th>$p^d\phi^{-1}(m/\phi(p^d))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>576</td>
<td>2 ${577, 629, 679, 969, 1095, 1365, 873, 1071}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= {1154, 1258, 1358, 1938, 2190, 2730, 1746, 2142}$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>288</td>
<td>4 ${323, 365, 455, 459, 555, 585}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= {1292, 1460, 1820, 1836, 2220, 2340}$</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>144</td>
<td>8 ${185, 219, 273, 285, 315}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= {1480, 1752, 2184, 2280, 2520}$</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>72</td>
<td>16 ${73, 91, 95, 111, 117, 135}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= {1168, 1456, 1520, 1776, 1872, 2160}$</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>36</td>
<td>32 ${37, 57, 63}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= {1184, 1824, 2016}$</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>18</td>
<td>64 ${19, 27}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= {1216, 1728}$</td>
</tr>
</tbody>
</table>

We have thus obtained all the elements of $\phi^{-1}(576)$. Arranging these in order, we can record them in our table.

It is noteworthy that in our calculations, the even elements of sets recorded earlier, play no role.

*Note*: Let $C_0(x)$ denote the set of odd and $C_e(x)$ that of even elements of $\phi^{-1}(x)$, then we leave it to the reader to show that $C_e(2m) = 2(C_0(2m) \cup C_e(m))$. 
The importance of this observation will be realized in the next section.

5. **The Number of Solutions of the Equation \( \phi(x) = m \)**

For any given \( m \), let \( v_0(m) \) and \( v_e(m) \) denote respectively the number of the odd and the even solutions of the equation

\[
\phi(x) = m. \tag{5.1}
\]

Then from the example in the preceding section, it will be clear that for \( m = 2^km_o \), where \( m_o \) is an odd number \( \geq 1 \), we have

\[
v_e(2^km_o) = v_o(2^km_o) + v_o(2^{k-1}m_o) + \ldots + v_o(2m_o) + v_o(m_o)
= v_o(2^km_o) + v_o(2^{k-1}m_o), \quad k \geq 1. \tag{5.2}
\]

For \( k = 0 \), we have

\[
v_o(m_o) = 0 = v_o(m_o), \quad m_o \geq 3;
v_e(1) = 1 = v_o(1).
\]

Here, we must state that there is no method of finding \( v_o(m) \) except by actual computation, as explained in the preceding section. For \( m = 2^k \), we have, however, the following:

**Theorem 2** — \( v_o(2^k) = 1 \), if \( 0 \leq k \leq 31; \)

\[
= 0, \text{ otherwise.}
\]

The proof depends on a well-known property of Fermat’s numbers.

**Proof:** The divisors of \( 2^k \) are

\[
1, 2, 4, \ldots, 2^k.
\]

The only values of \( j \) for which \( 2^j + 1 \) is a prime are

\[
j = 0, 1, 2, 4, 8, 16.
\]

Since \( 2^k \) has no prime divisor other than 2, any odd number \( n \) for which

\[
\phi(n) = 2^k
\]

must be a product of distinct odd primes of the form \( 2^j + 1 \). The theorem is true for \( k = 0 \), and every integer from 1 to 31 has a unique partition into the elements 1, 2, 4, 8, 16. Hence the first part of the theorem follows. The second part also follows if we accept that the Fermat numbers \( 2^{2n} + 1 \) are all composite for \( n \geq 5 \). In case this conjecture is untrue, the theorem will have to be stated in the form

\[
v_o(2^k) = 1 \text{ or } 0
\]
for all values of \( k \) including 0. In particular it is zero for \( k = 32 \) and 1 for each \( k \leq 31 \).

**Example** — The only odd solution of the equation

\[
\phi(x) = 2^{2^9}
\]

is \( x = (2^{16} + 1) (2^8 + 1) (2^4 + 1) (2 + 1) \).

6. **The Inequality** \( \phi(x) \leq m \)

In this section we assume that \( m \) is not too small.

Let \( V_o(m) \) and \( V_e(m) \) denote respectively the numbers of odd and even solutions of the inequality

\[
\phi(x) \leq (m).
\]

Then from (5.2), we immediately have

\[
V_e(m) + V_o(2m) = V_e(2m).
\] ... (6.1)

From our tables it appears that real numbers \( \alpha \) and \( \beta \) exist such that

\[
V_o(x) \approx \alpha x \text{ and } V_e(x) \approx \beta x
\]

(\( \approx \) means approximately equal to).

Assuming this to be true, (6.1) will give

\[
\beta x + 2\alpha x \approx 2\beta x.
\]

Hence

\[
\beta \approx 2\alpha.
\]

This means that the number of even solutions of \( \phi(x) \leq m \) is about twice the number of its odd solutions.

Tables show that

\[
\alpha \approx 0.648; \text{ and } \beta \approx 1.295
\]

**Acknowledgement**

The author must thank Messrs Ajeet Singh and Nirmal Roberts for their kind help. Also J. C. Parnami, using a well-know Lemma, was able to show that \( \alpha, \beta \) do exist and

\[
\alpha = \frac{1}{3} \prod_p \left( 1 + \frac{1}{p(p-1)} \right)
\]

where \( p \) runs over all primes.
A specimen page from the Table of values of $V_o, V_a$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$V_o$</th>
<th>$V_a$</th>
<th>$m$</th>
<th>$V_o$</th>
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